SPORT TK-EuroAmerican Journal of Sport Sciences

https://revistas.um.es/sportk Online ISSN: 2340-8812

Received: **01/02/2023.** Accepted: **27/04/2023.** Published: **17/04/2024**

Equilibration of athletes' body state in the course of increasing the training intensity

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ABSTRACT

Recovery of an athlete after injury allows to achieve either the previous results or relatively equal. Moreover, the prescribed drugs or recovery exercises are offered in courses without the possibility of process adjustments. The relevance of the study is determined primarily by a large number of universal methods of restoration, if necessary, development of more patient-specific approach. The novelty of the study is determined by the fact that the human body is a dynamic system, the state of which is different at any time. The authors developed the idea of a stochastic process problem in the form of a system with a variable state, which can be utilized to generate a process, in order to build a technique for recognizing and predicting the state of dynamic systems. The authors showed that the dynamic system of the body can be considered not only as a process element, but also as a structure that allows to fully achieve recovery during the training period of athletes and their off-season indicators. The paper has developed a model that allows to predict the state of the dynamic system and harmonise the load and the recovery process of the athlete's body. The developed method for identifying and predicting the state of dynamical systems with the improvement of the method of covariance functions factorisation makes it possible to determine the main matrices of a dynamical system using the Riccati equation.

KEYWORDS

Recovery; Organism; Dynamic System; Athletic Performance

1. INTRODUCTION

Nowadays, many researchers devote much attention to the problem of recovery, which is of great applied importance, including for the achievement of strong performance (Skorski et al., 2022; Hamlin et al., 2021). It should be emphasised that strenuous and prolonged physical activity is necessarily accompanied by one or another degree of fatigue, which, in turn, causes recovery processes, stimulates adaptive restructuring in the body. The correlation between the processes of fatigue and recovery is, in fact, the physiological basis of the process of sports training. As it is known, recovery processes in the athlete’s body are the most important biochemical, physiological and mental processes, the essence of which is that after muscle activity, reverse changes occur in the work of functional systems that ensured the completion of the exercise. The results of a comparative analysis of the data provided by different authors, who studied the factors influencing the reliability of the competitive technique in recovery, made it possible to identify one common point. Fatigue during a competitive sparring was identified as the main factor (Chang, 2020). Changing the individual stamp of performing a technical action in sports, which occurs in the event of physical fatigue, and, accordingly, a decrease in the special performance of an athlete, will lead to a violation of the usual structure of the exercise, which in turn will significantly reduce the possibility of its performance in a real fight.

From the foregoing, it can be seen that in order to maintain a high level of special performance during the competition, it is necessary to optimise the processes of recovery after loads. For this purpose, it is necessary to study the peculiarities of the course of recovery processes in the athlete’s body, taking into account the energy supply of training and competitive activity and recovery after injury. Based on the knowledge of the mechanisms and sources of energy supply of the athletes' body potential, from the standpoint of the effective time of work in the zones of physiological power, three ranges of time for the performing an exercise were identified: up to 15 s, from 15 to 30 s, more than 30 s (Gera et al., 2018). The exact distribution of the time intervals required for the maximum implementation of the motor potential makes it possible to create target programs of training sessions aimed at improving the special mechanisms of energy supply (Fraenkel, 1931).

Energy demands are distributed as follows, 62% of the time of competitive situations pass in the zone of anaerobic-alactic energy supply - the zone of rather difficult training, 28% of the time falls on the glycolytic zone of energy supply (Zelenin et al., 2016). In this regard, the organisation of training programs for athletes is a complex process, in which, together with the tasks of technical and tactical readiness, an important place is given to the tasks of the formation of speed and power performance of specific motor actions in conditions of limited time, maximum mobilisation of psychophysiological functions and readiness for an instant transition (Bansbach et al., 2017). A comparative analysis of the significance of individual metabolic factors in different sports is performed. The highest percentage of glycolytic anaerobic capacity was observed in judokas (20.9%), and judokas were second only to basketball players in terms of glycolytic anaerobic power (27.4%) (Southerland et al., 2017). Accordingly, sports achievements are most dependent on the level of development of anaerobic capabilities (Diez et al., 2017).

Studies have also shown that competitive sparring can be characterised as a glycolytic anaerobic load, which leads to significant shifts in the acid-alkali balance in the blood (Büttner et al., 2020). According to the results of the model experiment, a probable positive effect of training loads of an anaerobic glycolytic nature on the change in individual indicators of competitive activity was revealed (Thomas et al., 2014). In addition, it was found that the maximum growth rates of indicators of competitive activity are observed when performing loads of an anaerobic glycolytic nature in the range from 40 to 48% of the total value of the training load (Hewett et al., 2019). The characteristics of the power and capacity of the anaerobic lactic system of fighters, especially older ones, are closely related to the indices of their special endurance (Wares et al., 2015). Athletes who had the highest levels of these indicators ranked highest when comparing them according to manifestations of special work capacity (Rabiei et al., 2018).

It should also be noted that in the process of forming the body adaptation to physical activity after injuries, the course of recovery processes improves, its effectiveness increases. In untrained individuals, the recovery period is lengthened, and the “supercompensation” phase is poorly expressed. Highly qualified athletes have a short recovery period and more significant phenomena of "supercompensation". Thus, knowing the leading mechanism of energy supply and the main patterns of recovery processes after injury, we will be able to identify the main factors that limit the effectiveness of recovery processes in athletes, which will allow us to purposefully and selectively influence them. The fact that the human body is a dynamic system with changing states at all times determines the study's uniqueness. The authors demonstrate how the body's dynamic system may be viewed not only as a process component but also as a structure that enables athletes and their off-season markers to fully achieve recovery.

2. METHODS

It is necessary to introduce the idea of a stochastic process problem in the form of a system with a changeable state that can be utilized to construct a process in order to develop a method for recognizing and forecasting the state of dynamical systems. The goal function and constraints are both given stochastic formulations by stochastic programming problems. It is natural to define an optimal strategy in the form of a deterministic vector in stochastic programming problems corresponding to circumstances in which a decision should be made before observing realization of random conditions and it is impossible to correct a decision upon obtaining information about realized values of random parameters.

A dynamic system that generates some random process, in linear representation, can be specified by five matrices $A(t), B(t), C(t), Q, P_i$, system of differential and algebraic equations, with given initial conditions (1)–(2):

$$\frac{d\vec{x}}{dt} = A(t)\vec{x}(t) + B(t)\vec{u}(t), T_i \leq t \tag{1}$$

$$\vec{y}(t) = C(t)\vec{x}(t), T_i \leq t \tag{2}$$

where $\vec{x}(t)$ – state vector with dimensionality $(n \times 1)$; $\vec{u}(t)$ – white exciting process with dimensionality $(p \times 1)$; $\vec{y}(t)$ – observable process with dimensionality $(m \times 1)$.

Equation (1) is a linear equation of state and (2) is an observation equation. The matrices $A(t), B(t), C(t)$ have corresponding dimensionalities $(n \times n), (n \times p), (m \times n)$. The input process $\vec{u}(t)$ (it can be noises of the body, thermal drops in the elements of the body) has a covariance function of the form (3):

$$E\left[\vec{u}(t)\vec{u}^T(\tau)\right] = Q\delta(t - \tau) \tag{3}$$

The initial state vector is a random variable with a covariance matrix (4):

$$E\left[\vec{x}(T_i)\vec{x}^T(T_i)\right] = K_x(T_i, T_i) = P_i \tag{4}$$

where E – the expectation operator.

Many results of the theory of random processes can be expressed in terms of the matrix of covariance functions (5):

$$K_y(t, \tau) = E\left[\vec{y}(t), \vec{y}^T(\tau)\right] \tag{5}$$

To solve the problem of predicting the state of dynamic systems, it is necessary to create signal libraries, that is, to solve the direct problem; therefore, the procedure for finding the covariance function of a random process should be performed by its representation in state variables. We will give the definition of the covariance function of a random initial process $\vec{y}(t)$ from its description in state variables without proof and use for our purposes the solution of the inverse problem - finding the state variables from a known covariance matrix. According to (2) $K_y(t,\tau)$ it is easy to associate the state vector (6) with the covariance matrix $\vec{x}(t)$ (6):

$$K_y(t,\tau) = C(t) K_x(t,\tau) C^T(\tau) \tag{6}$$

The covariance matrix $K_x(t,\tau)$, in turn, satisfies the differential equation (7):

$$\dot{K}_x(t,\tau) = A(t) K_x(t,t) + K_x(t,t) A^T(t) + B(t) Q B^T(t) \tag{7}$$

with the initial condition (8):

$$K_x(T_i, T_i) = P_i \tag{8}$$

Due to the uncorrelatedness of $\vec{u}(t')$ and $\vec{x}(t)$ on the integration interval of the differential equation (7), we obtain (9):

$$K_x(t,\tau) = \begin{cases} \Theta(t,\tau) K_x(\tau,\tau), t \geq \tau \\ K_x(t,t) \Theta^T(\tau,t), \tau \geq t \end{cases} \tag{9}$$

where $\Theta(t,\tau)$ – transition matrix obtained from the differential equation (10)–(11):

$$\dot{\Theta}(t,t_0) = A(t) \Theta(t,t_0) \tag{10}$$

$$\Theta(t_0,t_0) = I \tag{11}$$

where I – identity matrix.

The solution for the state vector $\vec{x}(t)$ at any moment of time has the form (12):

$$\vec{x}(t) = \Theta(t,t_0) x(t_0) \tag{12}$$

Property (9) is valid for all processes represented by state variables in the form (1) - (5). For the stationary case, if the parameters of the system that generate the process $\vec{y}(t)$ are constant, then the transition matrix is determined by an exponential factor (13):

$$\Theta(t,\tau) = e^{A(t-\tau)} \tag{13}$$

In order for the matrix $K_x(t,t+\Delta t)$ to be a function of only Δt, the matrix $K_x(t,t)$ must, according to (9), be equal to a constant value P_∞. This constant matrix is a stationary solution to equation (7). Therefore, it is possible to simulate the segments of a stationary process using systems with constant parameters and setting the covariance matrix of the initial state P_i equal to P_∞. As shown, the stationary solution of equation (7) has the form (14):

$$P_\infty = \int_0^\infty e^{At} BQe^{A^T t} dt = \frac{1}{2\pi j} \int_{-j\infty}^{j\infty} \left[Is - A\right]^{-1} BQB^T \left[-Is - A^T\right]^{-1} ds \tag{14}$$

and the covariance matrix of the state vector (15):

$$K_x(t,t+\Delta t) = \begin{cases} e^{-A\Delta t} P_\infty, \Delta t \le 0 \\ P_\infty e^{A\Delta t}, \Delta t > 0 \end{cases} \tag{15}$$

and hence, according to formula (6), we obtain (16):

$$K_y(t,t+\Delta t) = \begin{cases} C(t) e^{-A\Delta t} P_\infty C^T(t), \Delta t \le 0 \\ C(t) P_\infty e^{A^T \Delta t} C^T(t), \Delta t > 0 \end{cases} \tag{16}$$

The correlation function of the system output is expressed in terms of the system state variables and, therefore, on the contrary, there should be an inverse problem: knowing the output correlation matrix for given $C(t)$ and P_∞=const, determine the structure of the system function, that is, solve the problem of partial identification and system forecasting. To solve this problem, it is advisable to use the method of covariance functions factorisation. Earlier it was assumed that there is a description of a random process in state variables, and a model was proposed for determining the covariance function (matrix) of this process.

3. RESULTS AND DISCUSSION

Based on the above, now we will consider the situation when the covariance function of the process is known, and we will give a model for describing the process in state variables. Let there be the covariance matrix of the system output $\vec{y}(t) - K_y(t,\tau)$, built on the basis of the observation of a random output process $\vec{y}(t)$ measured by the device with the observation matrix $C(t)$ on the interval $T_i \le t, \tau \le T_j$. For the process under consideration $\vec{y}(t)$ we use its description in state variables (1) - (4), for which it is necessary to find the matrices $A(t), B(t), C(t) Q$ and P_i, and, that is, to solve the problem of factorisation of the covariance function. The factorisation problem can be solved in the time domain or in the frequency domain. To solve the factorisation problem in the frequency domain, it is necessary to find the Fourier transform of (16). Since the only way to take into account the possible nonstationarity of the system's output process is factorisation in the time domain, we will consider its capabilities. We obtain (17):

$$K_y(t,\tau)=\begin{cases} C(t)\Theta(t,\tau)K_x(\tau,\tau)C^T(\tau), t\geq\tau \\ C(t)K_x(t,t)\Theta^T(\tau,t)C^T(\tau), \tau\geq t \end{cases} \tag{17}$$

From the properties of transition matrices (18):

$$\left(\dot{\Theta}(t,t_0)=A(t)\Theta(t,t_0)\right) \tag{18}$$

it follows that the covariance function of a process $\vec{y}(t)$, generated by a system with an unknown structure, represented in the form of state variables, must have a separable (decomposed) form (19):

$$K_y(t,\tau)=\begin{cases} F^T(t)G(\tau), t\geq\tau \\ G^T(t)F(\tau), \tau\geq t \end{cases} \tag{19}$$

Where (20)–(21):

$$F^T(t)=C(t)\Theta(t,t_1) \tag{20}$$

$$G(t)=\Theta(t_1,t)K_x(t,t)C^T(t) \tag{21}$$

t_1 – an arbitrary time variable in the process definition domain $\vec{y}(t)$ (take it equal to T_i).

Dimensionalities $F(t)$ and $G(t)$ are related to the dimensionality of the output process $\vec{y}(t)$. Therefore, the first step in the factorisation problem is the search for $(n\times m)$-dimensional matrices $F(t)$ and $G(t)$ and given covariance functions. The second step is to develop an algorithm for obtaining $A(t),B(t),C(t),Q$ and P_i from $F(t)$ and $G(t)$. Consider the case when no restriction is imposed on the minimality n. In the case of non-minimal factorisation, each element of the covariance matrix must have the form (22):

$$\left[K_y(t,\tau)\right]_{ij}=\begin{cases} \sum_k^{n'}\sum_l^{n'} b_{ij}^*(k,l)\, f_k(t)q_i(\tau), T_i\leq\tau\leq t\leq T_j \\ \sum_k^{n'}\sum_l^{n'} b_{ij}^*(k,l)q_i(t)\, f_k(\tau), T_i\leq t\leq\tau\leq T_j \end{cases} \tag{22}$$

The members of the series $\left\{f_k\left(t\right),1\le k\le n'\right\},\left\{q_i\left(\tau\right),1\le l\le n'\right\}$ will be $n=2$, for example, if (23):

$$K_y\left(t,\tau\right)=\begin{bmatrix} P_1e^{-k_1|t-\tau|} & 0 \\ 0 & P_2e^{-k_2|t-\tau|}\end{bmatrix} \tag{23}$$

hence (24):

$$\begin{matrix} f_1\left(t\right)=e^{-k_1t} & q_1\left(\tau\right)=e^{k_1\tau} & b_{11}\left(k,l\right)=P_1\delta_{k_1}\delta_{i_1} \\ f_2\left(t\right)=e^{-k_2t} & q_2\left(\tau\right)=e^{k_2\tau} & b_{22}\left(k,l\right)=P_2\delta_{k_2}\delta_{i_2} \\ & & b_{12}\left(k,l\right)=b_{21}\left(k,l\right)=0\end{matrix} \tag{24}$$

In general, the column-vector (25):

$$\vec{f}\left(t\right)=\begin{pmatrix} f_1\left(t\right) \\ \vdots \\ f_n\left(t\right)\end{pmatrix},\vec{q}\left(t\right)=\begin{pmatrix} q_1\left(t\right) \\ \vdots \\ q_n\left(t\right)\end{pmatrix} \tag{25}$$

Form the systems of linearly independent functions. Each element of matrix (23) can be written in the form (26):

$$\left[K_y\left(t,\tau\right)\right]_{ij}=\vec{f}^{\,T}\left(t\right)\begin{bmatrix} g_{ij}^*\left(1,1\right) & g_{ij}^*\left(1,2\right) & \dots & g_{ij}^*\left(1,n^1\right) \\ \vdots & & \ddots & \vdots \\ g_{ij}^*\left(n^1,1\right) & & \dots & g_{ij}^*\left(n^1,n^1\right)\end{bmatrix}\vec{q}\left(\tau\right)=\vec{f}^{\,T}\left(t\right)G_{ij}^*\vec{q}\left(\tau\right) \tag{26}$$

where G_{ij}^* – the dimensionality matrix $n^1\times n^1$.

Using this representation, it is possible to factorise, not necessarily of the minimum degree, we obtain in the form of matrices of dimensions $\left(\left(n^1m\right)\times m\right)$ (27)–(28):

$$F^*\left(t\right)=\begin{bmatrix} \vec{f}\left(t\right) & 0 & \dots & 0 \\ 0 & \vec{f}\left(t\right) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \vec{f}\left(t\right)\end{bmatrix} \tag{27}$$

$$G^*\left(t\right)=\begin{bmatrix} G_{11}^*\vec{q}\left(t\right) & G_{12}^*\vec{q}\left(t\right) & \dots & G_{1n}^*\vec{q}\left(t\right) \\ G_{21}^*\vec{q}\left(t\right) & G_{22}^*\vec{q}\left(t\right) & & \\ \vdots & \vdots & \ddots & \vdots \\ G_{m1}^*\vec{q}\left(t\right) & 0 & \dots & G_{mn}^*\vec{q}\left(t\right)\end{bmatrix} \tag{28}$$

A sufficient criterion for verifying that the matrices $F^*(t)$ and $G^*(t)$ are factors of the minimum degree is the positive definiteness of two matrices (29)–(30):

$$M_F = \int_{T_i}^{T_j} F^*(t) F^{*T}(t) dt \tag{29}$$

$$M_G = \int_{T_i}^{T_j} G^*(t) G^{*T}(t) dt \tag{30}$$

that is, the rank of matrices M_F and M_G is equal to the minimum dimensionality $\left(n^* = n^1 m\right)$. Since M_F and M_G re symmetric and positively correlated, they must be consistent through non-degenerate transformations T_F and T_G in accordance with matrices - conditional mathematical expectations E_F and E_G. There is always a coordinate system in which random variables are uncorrelated, and the new system is associated with an output linear transformation, that is (31)–(32):

$$T_F E_F T_F^T = M_F \tag{31}$$

$$T_G E_G T_G^T = M_G \tag{32}$$

and in this case (33)–(34):

$$\int_{T_i}^{E_j} \left(T_F E_F T_F^{-1} F^*(t) - F^*(t)\right)\left(T_F E_F T_F^{-1} F^*(t)\right)^T dt = 0 \tag{33}$$

$$\int_{T_i}^{T_j} \left(T_G E_G T_G^{-1} G^*(t) - G^*(t)\right)\left(T_G E_G T_G^{-1} G^*(t)\right)^T dt = 0 \tag{34}$$

As a result, we obtain (35)–(37):

$$F^*(t) = T_F E_F T_F^{-1} F^*(t), T_i \le t \le T_j \tag{35}$$

$$G^*(t) = T_G E_G T_G^{-1} G^*(t), T_i \le t \le T_j \tag{36}$$

$$K_y(t,\tau) = F^{*T}(t) G^*(\tau) = F^{*^T}(t) T_F^{-1^T} E_F T_F^T T_G E_G T_G^{-1} G^*(\tau), t > \tau \tag{37}$$

To obtain the minimum degree factor for (19), we define $F(t)$ and $G(t)$ as follows (38)–(39):

$$F(t) = N_1 T_F^{-1} F^*(t) \tag{38}$$

$$G(t)=N_2T_G^{-1}G^*(t) \tag{39}$$

where matrices N_1 and N_2, with dimensionality $(n\times n^*)$, satisfying the condition (40):

$$E_FT_F^TT_GE_G=N_1^TN_2 \tag{40}$$

and the matrix $E_FT_F^TT_GE_G$ has corresponding dimensionality $(n^*\times n^*)$. Now, having an algorithmic procedure for decomposing the covariance matrix $K_y(t,\tau)$ into factors, it is necessary, on the basis of $F(t)$ and $G(t)$ to determine the state matrices of the dynamic system $A(t),B(t),C(t)$ and the covariance matrices Q and P_i. Since the coordinate system of the state vector is not unified and it follows from this that these matrices are certainly not unified. Indeed, with the exception of its dimensionality, the matrix $A(t)$ is undefined. First, consider an implementation with a triplet matrix $(0,B^*(t),C^*(t))$, and then - questions about the transformation to coordinate systems with the desired properties (here "0" is a zero matrix $A^*(t)$). The transition matrix associated with $A^*(t)=0$ is the identity matrix. Therefore, according to (18), we have (41):

$$K_y(t,\tau)=\begin{cases}C_*(t)K_{x*}(\tau,\tau)C_*^T(\tau),t>\tau\\ C_*(t)K_{x*}(t,t)C_*^T(\tau),\tau>t\end{cases} \tag{41}$$

Or (42):

$$F^T(t)=C_*(t),G(t)=K_{x*}(t,t)C_*^T(\tau) \tag{42}$$

Note that (42) does not define $K_{x*}(t,t)$) uniquely, since the matrix $C_*(t)$ has the dimensionality $(m\times n)$, and the matrix $K_{x*}(t,t)$ has the dimensionality $(n\times n)$. Using (7), one can show the validity of the following two properties (43)–(44):

$$\dot{K}_{x*}(t,t)=B_*(t)QB_*(t) \tag{43}$$

$$F^T(t)\dot{G}(t)-G^T(t)\dot{F}(t)=C_*(t)B_*(t)QB_*^T(t)C_*(t) \tag{44}$$

It is known that the covariance function of the derivative of a random vector $\vec{y}(t)$ is determined by expression (45):

$$K_{\dot{y}}(t,\tau)=\frac{\partial^2}{\partial t\,\partial\tau}K_y(t,\tau) \tag{45}$$

Or (46):

$$K_y(t,\tau)=\begin{cases}\dot{F}^T(t)\dot{G}(\tau),t>\tau\\ \dot{G}^T(t)\dot{F}(\tau),\tau>t\end{cases}+\left(T^T(t)\dot{G}(t)-G^T(t)\dot{F}(t)\right)\delta(t-\tau) \tag{46}$$

If the process is differentiable in the mean square, then the coefficient at the δ-function should be equal to zero. Noting that Q can be considered positive definite without loss of generality, we come to the conclusion that for a process that differentiates in the mean square (47):

$$C_*(t)B_*(t)=0 \tag{47}$$

For a differentiable process, the expansion, according to (46), has the form (48)–(49):

$$F_{\dot{y}}^T(t)=\dot{F}^T(t)=\dot{C}_*(t) \tag{48}$$

$$G_{\dot{y}}(t)=\dot{G}(t)=K_{x*}(t,t)\dot{C}_*^T(\tau) \tag{49}$$

It follows that the implementation for the derived process would be a function $\left(0,B_*(t),C_*(t)\right)$. The inference strategy is to repeat this procedure, $\vec{y}^l(t)$ – the derivative of the differentiation process until a higher order of $\vec{y}(t)$ is reached, which still exists in the root-mean-square sense. The need for this procedure is that the decomposition of the process $\vec{y}(t)$ and all its derivatives to $\vec{y}^l(t)$ it is necessary to determine the state matrices. In the general case, for the interval $1\le k\le l$ we have (50)–(51):

$$F^{(k-1)^T}(t)G^{(k)}(t)-G^{(k-1)^T}(t)F^{(k)}(t)=0 \tag{50}$$

$$C_*^{(k-1)}(t)B_*(t)=0 \tag{51}$$

The l-th-order derivative has the representation $\left(0,B_*(t),C_*^{(l)}(t)\right)$ and expansion $C_*^{(l)}(t)$ and $K_{x^*}(t,t)C_*^{(l)}(t)$ for $F^T(t)$ and $C(t)$, respectively.

Equations (48), (49), (50), (51) contain the main results connecting the differentiation of the process with the derivatives of the factors $F(t)$ and $G(t)$. Briefly, the algorithm for determining state matrices $\left(0,B_*(t),C_*(t)\right)$ can be reduced to the following sequence of operations. We arrange the components $K_y(t,\tau)$ in the reverse order of their differentiation, that is, the first components r_1 have derivatives of only zero order, the second components r_2 have derivatives of only first order, etc. It is also assumed that the matrix columns $F(t)$ and $G(t)$ are respectively interchanged and that $Q - m$ -dimensional identity matrix. Next, we partition the matrices $B_*(t)$ and $C_*(t)$ in accordance with the order of differentiation $(r_1, r_2, ..., r_L)$ (52)–(53):

$$B_*(t) = \left[B_{*1}(t) \;\; B_{*2} \cdots B_{*L}(t) \right] - n \text{ component part,} \tag{52}$$

$$C_*(t) = \begin{array}{c} \begin{bmatrix} C_{*1}(t) \\ C_{*2}(t) \\ \vdots \\ C_{*L}(t) \end{bmatrix} \end{array} \begin{array}{c} r_1 \\ r_2 \\ \vdots \\ r_L \end{array} - n \text{ component part} \tag{53}$$

Each component part $\vec{y}_L(t)$ has the representation $(0, B_*(t), C_{*l}(t))$, which is $(l-1)$-divisible (and not l-divisible) differentiated in the root-mean-square sense (Figure 1).

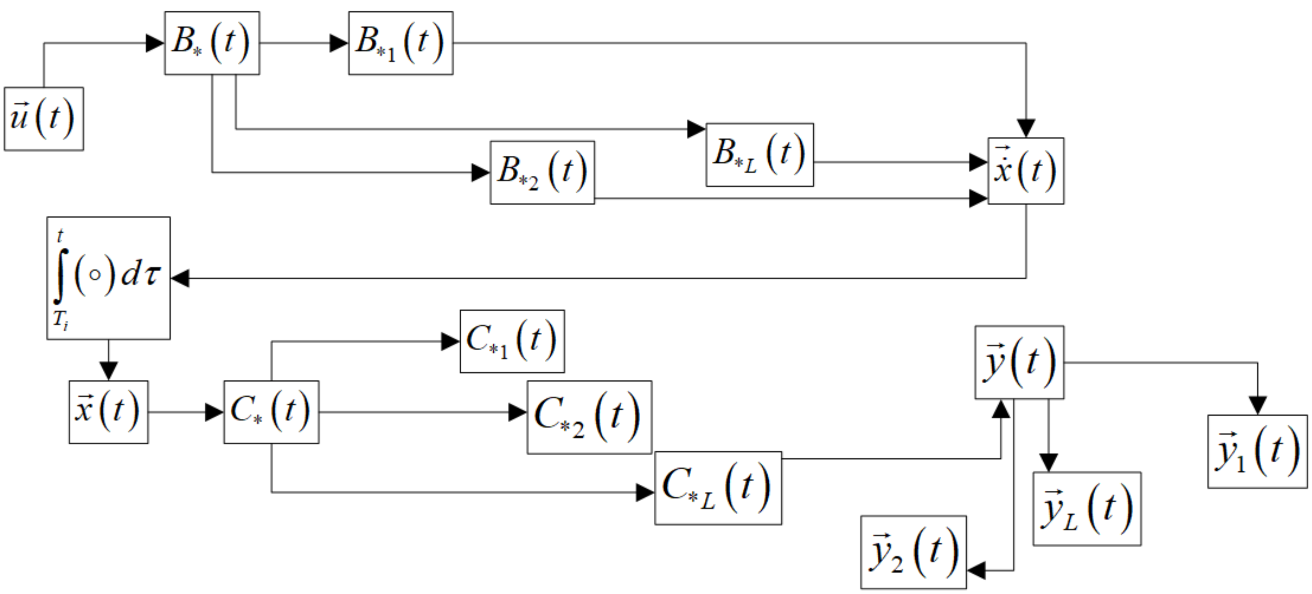

Figure 1. Block diagram of the initial process representation, respectively with the differentiation of its components

According to (43), the differential equation for the covariance function of the process is written in the form (54):

$$\dot{K}_{x*}(t,t) = B_*(t) B_*^T(t) = \sum_{l=1}^{L} B_{*l}(t) B_{*l}^T(t) \tag{54}$$

If we use (51) in the corresponding components $\vec{y}_i(t)$, then it follows from the indicated differentiation conditions that (55):

$$C_*^{(k)}(t)C_*(t)=0\,, \tag{55}$$

$$0\le k<l-1$$

Let us split $F(t)$ and $G(t)$ in the form (56)–(57):

$$F(t) = [F_1(t);\, F_2(t);\; F_L(t)] \tag{56}$$

$$G(t) = [G_1(t);\, C_2(t);\; G_L(t)] \tag{57}$$

Now, as is obvious from (41), (42), it is necessary to homologate $C_*(t)$ with $F^T(t)$, that is (58)–(59):

$$C_*(t)=F^T(t) \tag{58}$$

$$C_{*l}^T(t)=F_l(t) \tag{59}$$

In view of this, the matrices $F(t)$ and $C^T(t)$ can be freely mutually replaced in the following equations. Thus, we obtain (60)–(61)

$$G(t)=K_{x*}(t,t)F(t) \tag{60}$$

$$G_l(t)=K_{x*}(t,t)F_l(t) \tag{61}$$

Differentiating (61), we obtain (62):

$$\dot{G}_l(t)=\dot{K}_{x*}(t,t)F_l(t)+K_{x*}(t,t)\dot{F}_l(t)=B_*(t)B_*^T(t)C_{*l}^T(t)+K_{x*}(t,t)\dot{F}_l(t) \tag{62}$$

If $l=1$, then (63):

$$\dot{G}_1(t)=B_*(t)B_*^T(t)C_{*1}^T(t)+K_{x*}(t,t)\dot{F}_l(t) \tag{63}$$

If $l\neq 1$, then, taking into account (55), we obtain (64):

$$\dot{G}_1(t)=K_{x*}(t,t)\dot{F}_l(t) \tag{64}$$

After *l*-fold differentiation (65)–(66):

$$\dot{G}_l^{(l)}(t)=B_*(t)B_*^T(t)C_{*l}^{(l-1)T}(t)+K_{x*}(t,t)\dot{F}_l^{(l)}(t) \tag{65}$$

$$G_l^{(k)}(t)=K_{x*}(t,t)F_l^{(k)}(t),0\le k\le l-1 \tag{66}$$

Let us make the corresponding multiplication (65) by $C_{*l}^{(l-1)}(t)$, putting $C_{*l}^{(l-1)}(t)$ in front, and using the result of permutation (66), we obtain the matrix equation of dimensionality ($r_l \times r_l$) (67):

$$\left(C_{*l}^{(l-1)}(t)B_*(t)\right)\left(B_*^T(t)C_{*l}^{(l-1)T}(t)\right)=F_l^{(l-1)T}(t)G_l^{(l)}(t)-G_l^{(l-1)T}(t)\dot{F}_l^{(l)}=D_l(t) \tag{67}$$

which defines the matrix system $D_l(t)$. Suppose the matrix $K_y(t,\tau)$ is positive definite. From this assumption and from the conditions of differentiation it follows that $D_l(t)$ is a positive definite matrix; therefore, it has a positive definite square root. The matrix $\left(C_{*l}^{(l-1)}(t)B_*(t)\right)$ with dimensionality $(r_l \times m)$ can be expressed as (68):

$$\left(C_{*l}^{(l-1)}(t)B_*(t)=\begin{matrix} 0 & \dots & D_l^{\frac{1}{2}}(t) & \dots & 0 \\ r_l & \dots & r_l & \dots & r_l \end{matrix}\right) \tag{68}$$

Now we substitute the transposed matrix (68) into the formula (65), we obtain (69):

$$B_{*l}(t)\left[D_l^{\frac{1}{2}}(x)\right]^T=G_l^{(l)}(t)-K_{x*}(t,t)F_l^{(l)}(t) \tag{69}$$

Hence (70):

$$B_{*l}(t)=\left(G_l^{(l)}(t)-K_{x*}(t,t)F_l^{(l)}(t)\right)\left[D_l^{-\frac{1}{2}}(t)\right]^T \tag{70}$$

Formula (70) defines the expansion $B_*(t)$ through $K_{x*}(t,t), F(t)$ and $G(t)$; however $K_{x*}(t,t)$ remains unknown. We define the differential equation for $K_{x*}(t,t)$, substitute (70) in (54), we obtain (71):

$$\begin{gathered}\dot{K}_{x*}(t,t)=\left\{\left[G_1^{(1)}(t)G_2^{(2)}(t)\cdots G_L^{(L)}(t)\right]-K_{x*}(t,t)\left[F_1^{(1)}(t)F_2^{(2)}(t)\cdots F_L^{(L)}(t)\right]\right\}\times \\ \times\begin{bmatrix} D_1(t) & 0 & \cdots & 0 \\ 0 & D_2(t) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & D_L(t) \end{bmatrix}^{-1}\left\{\begin{matrix}\left[G_1^{(1)}(t)G_2^{(2)}(t)\cdots G_L^{(L)}(t)\right]- \\ -K_{x*}(t,t)\left[F_1^{(1)}(t)F_2^{(2)}(t)\cdots F_L^{(L)}(t)\right]\end{matrix}\right\}\end{gathered} \tag{71}$$

This is a Riccati type differential equation. In order to prove the existence of a well-defined solution, it is necessary to reduce it to an ordinary differential equation using a series of substitutions. Let us denote (72)–(73)

$$\begin{gathered}\left[G_1^{(1)}(t)G_2^{(2)}(t)\cdots G_L^{(L)}(t)\right]=\tilde{G}\\ \left[F_1^{(1)}(t)F_2^{(2)}(t)\cdots F_L^{(L)}(t)\right]=\tilde{F}\end{gathered} \tag{72}$$

$$\begin{bmatrix} D_1(t) & 0 & \cdots & 0 \\ 0 & D_2(t) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & D_L(t) \end{bmatrix}=\tilde{D} \tag{73}$$

we arrive at equation (74):

$$\dot{K}_{x*}(t,t)=\tilde{F}\tilde{F}^T K_{x*}^2(t,t)-\left(\tilde{D}\tilde{G}\tilde{F}^T+\tilde{D}\tilde{F}\tilde{G}\right)K_{x*}(t,t)+\tilde{D}\tilde{G}^2 \tag{74}$$

and introducing one more notation (75):

$$\begin{gathered}\tilde{F}\tilde{F}^T=f(t) \quad -\left(\tilde{D}\tilde{G}\tilde{F}^T+\tilde{D}\tilde{F}\tilde{G}\right)=q(t)\\ \tilde{D}\tilde{G}^2=h(t) \qquad K_{x*}(t,t)=z(t)\end{gathered} \tag{75}$$

We obtain (76):

$$\dot{z}(t)=f(t)z^2(t)+q(t)z(t)+h(t) \tag{76}$$

the classical Riccati equation, which is reduced by substitution (78):

$$\varphi(t)=exp\left(-\int fzdt\right) \tag{78}$$

into a nonzero solution to linear differential equation (79):

$$f\varphi''-(f'+fq)\varphi'+f^2h\varphi=0 \tag{79}$$

The converse, since $h\neq 0$, then each nonzero solution to equation (79) becomes transformation (80):

$$z(t)=\frac{\varphi'}{\varphi f} \tag{80}$$

is translated into the solution of the Riccati equation, which, in turn, allows us to estimate the state space vector $\vec{x}(t)$ and matrices $A(t),B(t),C(t)$. Denote (81):

$$f=a_0,(f'+fq)=a_1,f^2=a_2 \tag{81}$$

we write the Laplace transform by equation (79) in the form (82)–(83):

$$\left(a_0s^2-a_1s+a_2\right)\varphi=0 \tag{82}$$

$$r_{1,2} = \frac{a_1 \pm \sqrt{a_1^2 - 4a_0a_2}}{2a_0} \tag{83}$$

The general solution of equation (79) is $\varphi = k_1e^{r_1t} + k_2e^{r_2t}$, and equation (78) is according to (80), we obtain (84):

$$Z = K_{x*}(t,t) = \frac{k_1r_1e^{r_1t} + k_2r_2e^{r_2t}}{\left(k_1r_1e^{r_1t} + k_2r_2e^{r_2t}\right)\tilde{F}\tilde{F}^T} \tag{84}$$

and based on formulas (6), (58), (75), we have (85):

$$\begin{aligned} K_y(t,\tau) = C(t)K_x(t,\tau)C^T(\tau) = C_*(t)\frac{k_1r_1e^{r_1t} + k_2r_2e^{r_2t}}{\left(k_1e^{r_1t} + k_2e^{r_2t}\right)\tilde{F}\tilde{F}^T}C_*^T(\tau) = \\ = F^T(t)\frac{k_1r_1e^{r_1t} + k_2r_2e^{r_2t}}{\left(k_1e^{r_1t} + k_2e^{r_2t}\right)\tilde{F}\tilde{F}^T}F(\tau) \end{aligned} \tag{85}$$

To determine the initial conditions, the solution of the Riccati equation is sufficient to require that the initial condition $K_{x*}(T_i,T_j)$ be described by a negative definite symmetric matrix $F(t)$ and $G(t)$ and that their derivatives are finite and continuous, the covariance function is determined by relation (19) and for (89)–(90):

$$\tilde{F} = \left[F_1^{(1)}(t) \quad F_2^{(2)}(t) \cdots F_L^{(L)}(t)\right] \tag{89}$$

$$\tilde{G} = \left[G_1^{(1)}(t) \ \ G_2^{(2)}(t) \cdots G_L^{(L)}(t)\right] \tag{90}$$

was a positive definite matrix.

To determine the initial conditions, relations (66) can be expressed in the form of a system of L matrix equations of dimensionality $(n\times n)$ (91):

$$\begin{array}{ccccccccc} [G_1^0(t) & G_2^0(t) & \cdots & G_L^0(t)] = K_{x*}(t,t) & [F_1^0(t) & F_2^0(t) & \cdots & F_L^0(t)] \\ [0 & G_2^1(t) & \cdots & G_L^1(t)] = K_{x*}(t,t) & [0 & F_2^1(t) & \cdots & F_L^1(t)] \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & G_L^{L-1}(t)] = K_{x*}(t,t) & [0 & 0 & \cdots & F_L^{L-1}(t)] \end{array} \tag{91}$$

It is necessary to solve these equations for the moment in time T_i and with respect to $K_{x*}(T_i,T_i)$. Perhaps in another way, using the method of pseudo-inversion of matrices for determination of $K_{x*}(T_i,T_i)$, combining all equations (91) in $(n\times n \ \ L)$-dimensional system.

Equation (68) for the given initial conditions simulates the desired solution for $K_{x*}(t,t)$. This, in turn, determines $B_*(t)$ through (69), and $C_*(t)$ directly determines using $F^T(t)$. Now we have a procedure for defining state variables. It is desirable to have a solution implementation of the system with constant parameters. Let us consider under what conditions this is possible.

Introduce into consideration the matrix $T(t)$, that defines the linear one-to-one transformation of the state vector $\vec{x}(t)$, differentiate (92):

$$\vec{\dot{x}}(t) = T(t)\vec{x}(t) \tag{92}$$

The state matrices for the transformed state vector have the form (93)–(97):

$$\dot{A}(t) = \left(T(t)A(t) + \dot{T}(t)\right)T^{-1}(t) \tag{93}$$

$$\dot{B}(t) = T(t)B(t) \tag{94}$$

$$\dot{C}(t) = C(t)T^{-1}(t) \tag{95}$$

$$\dot{Q} = Q \tag{96}$$

$$\dot{P}_i(t) = T(T_i)P_iT^T(T_i) \tag{97}$$

Let us determine the conditions under which it is possible to carry out such a transformation so that all state matrices are constant, that is, we find conditions when the matrices $\dot{A}(t), \dot{B}(t), \dot{C}(t)$ are constant, and the initial matrices are $\left(0, B_*(t), C_*(t)\right)$. From (93) at (98):

$$A(t) = 0 \tag{98}$$

and (99):

$$\dot{A}(t) = A_c \tag{99}$$

obtain (100):

$$\dot{T}(t) = A_cT(t) \tag{100}$$

where A_c – the matrix to be determined. The general solution to equation (100) has the form (101):

$$\dot{T}(t) = e^{A_c(t-T_i)}T(T_i) \tag{101}$$

at (102):

$$\dot{B}(t) = B_c \tag{102}$$

and (103):

$$B(t) = B_*(t) \tag{103}$$

after substituting (101) into (94) and differentiating, we find that to obtain a constant representation, it is necessary that the matrix $B_*(t)$ satisfies equation (104):

$$\dot{B}_*(t) = -B_*(t)\left(T^{-1}(T_i)A_cT(T_i)\right)B_*(t) = -A_TB_*(t) \tag{104}$$

where the matrix A_T is explicitly defined. We find (105) in a similar way:

$$\dot{C}_*(t) = \left(T^{-1}(T_i)A_cT(T_i)\right) = C_*(t)A_T \tag{105}$$

Therefore, for the existence of a system representation with constant (time-independent) parameters, it is necessary and sufficient for a matrix A_T to exist, which satisfies Eqs. (104) and (105). Then we have a representation through the transformation of the general solution (101). The triplet of representation matrices takes the form

$$\left(T(T_i)A_cT^{-1}(T_i), T(T_i)e^{A_c(t-T_i)}B_*(t), C_*(t)e^{-A_c(t-T_i)}T^{-1}(T_i)\right) \tag{106}$$

Although equations (104) and (105) form a necessary and sufficient criterion for the existence of a representation with constant parameters, this criterion is rather difficult to use. Simpler criteria for the case of differentiation of the first and second order will be presented with the simulation results below.

For the created method for identifying and predicting the state of dynamic systems in time, a method has been developed for determining the matrices of a dynamic system using the method of factorisation of covariance functions (solution of direct and inverse problems). According to the results of studies by various authors, it can be argued that anaerobic glycolysis is the leading mechanism of energy supply in the training and competitive activity of athletes (Hou, 2005). However, for a more complete understanding of the peculiarities of the course of recovery processes in the athlete’s body, it is necessary to dwell on the main regularities of the recovery processes after injuries (Wan et al., 2013).

Most researchers identify the following basic physiological patterns of recovery processes: unevenness, heterochronism, phase nature of recovery, selectivity of recovery, and the ability to train recovery processes (Weerasekara et al., 2018). Taking into account the unevenness and heterochronism of recovery processes, according to various literature sources, lactic acid utilisation takes from one to two hours (Pan et al., 2015). According to the phase of recovery in the competitive activity of athletes, we can say the following: taking into account the small intervals between competitions and the accumulation of fatigue after each subsequent attempt, it can be assumed that the recovery processes in the body of athletes on the day of the competition will correspond to the phase of reduced performance (Li et al., 2004). This further emphasises the importance of correcting recovery processes in the body of athletes (Musienko et al., 2013).

The selectivity of restorative processes is manifested in a different nature of the effect on individual functions of the body and various links of energy metabolism (Nagai et al., 2016). Understanding of the selective nature of training and competitive loads, as well as the selective nature of recovery, makes it possible to purposefully and effectively control the motor apparatus, autonomic functions and energy metabolism (Sun et al., 2007). After work with a predominantly aerobic orientation, the recovery of external respiration parameters, the phase structure of the cardiac cycle, and functional resistance to hypoxia occur more slowly than after anaerobic loads (Sielski et al., 2018). This feature can be traced both after individual training sessions, and after weekly microcycles. Therefore, it is logical, based on knowledge about the main mechanism of energy supply in athletics that the essence of recovery processes after competitive and training activities mainly consists in the utilisation of lactic acid, elimination of acidosis, restoration of energy substrates - glucose and glycogen, and in the restoration of the work of glycolytic enzymes (Condo et al., 2022).

The Riccati equation can be used to find the primary matrices of a dynamical system utilizing the newly created approach for identifying and predicting the state of dynamical systems with the improvement of the method of covariance functions factorisation. By using the substitution approach, this equation may be reduced to a linear differential equation, the solution of which can be used to solve the Riccati equation and identify the fundamental matrices of the dynamical system. The Riccati equation can be solved by using the state space technique, which makes it easier to understand the structure of dynamic systems by using more details about the random processes they generate. The employed method enables us to reduce the problem of predicting the structure of a dynamic system to the problem of predicting the structure with constant matrices of the state space, based on one-to-one differential transformations of the state vector.

4. CONCLUSIONS

The developed method for identifying and predicting the state of dynamical systems with the improvement of the method of covariance functions factorisation makes it possible to determine the main matrices of a dynamical system using the Riccati equation. By the substitution method, this equation is reduced to a linear differential equation, the solution of which is transformed into a solution of the Riccati equation, by solving which it is possible to determine the basic matrices of the dynamical system. The use of the state space method allows to reduce the problem of identifying the structure of a dynamic system using additional information about the random process generated by it for solving the Riccati equation. Based on one-to-one differential transformations of the state vector, the applied procedure allows us to reduce the problem of predicting the structure of a dynamic system to the problem of predicting the structure with constant matrices of the state space.

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AUTHOR CONTRIBUTIONS

All authors listed have made a substantial, direct and intellectual contribution to the work, and approved it for publication.

CONFLICTS OF INTEREST

The authors declare no conflict of interest.

FUNDING

This research received no external funding.

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