Augmented reality and GeoGebra 3D for improving spatial intelligence in teaching volumetric geometry

Realidad aumentada y GeoGebra 3D para mejorar la inteligencia espacial en la enseñanza de la geometría volumétrica

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Abstract

This study analyses the impact of augmented reality (AR) using GeoGebra 3D on teaching volumetric geometry and improving spatial intelligence in secondary school students. Spatial intelligence, fundamental in STEM areas, is developed through the visualisation and manipulation of three-dimensional objects, skills that are not fully developed by traditional teaching methods. A quasi-experimental design was implemented with an experimental group using GeoGebra 3D with AR and a control group using traditional methods. The sample consisted of high school students and included the PSVT:R test to measure spatial ability, academic tests and perceptual questionnaires. The results show a significant improvement in visualisation and spatial rotation skills in the experimental group, as well as an increase in student motivation and interest. AR facilitated interaction and understanding of complex geometric concepts, promoting more active and meaningful learning. In conclusion, the integration of GeoGebra 3D with AR improves spatial intelligence and optimises the teaching process of volumetric geometry, positioning itself as an effective and innovative technological tool in secondary education.

Keywords: Augmented reality, GeoGebra 3D, spatial intelligence, volumetric geometry, secondary education.

Resumen

Este estudio analiza el impacto de la realidad aumentada (RA) mediante la aplicación GeoGebra 3D en la enseñanza de la geometría volumétrica y en la mejora de la inteligencia espacial en estudiantes de secundaria. La inteligencia espacial, fundamental en áreas STEM, se desarrolla a través de la visualización y manipulación de objetos tridimensionales, habilidades que los métodos tradicionales de enseñanza no logran potenciar completamente. Se implementó un diseño cuasi-experimental con un grupo experimental, que utilizó GeoGebra 3D con RA, y un grupo control, que trabajó con métodos tradicionales. La muestra incluyó estudiantes de secundaria y se aplicaron el test PSVT:R para medir habilidades espaciales, pruebas académicas y cuestionarios de percepción. Los resultados muestran una mejora significativa en las habilidades de visualización y rotación espacial en el grupo experimental, así como un incremento en la motivación y el interés de los estudiantes. La RA facilitó la interacción y comprensión de conceptos geométricos complejos, favoreciendo un aprendizaje más activo y significativo. En conclusión, la integración de GeoGebra 3D con RA mejora la inteligencia espacial y optimiza el proceso de enseñanza de la geometría volumétrica, posicionándose como una herramienta tecnológica eficaz e innovadora en la educación secundaria.

Palabras clave: Realidad aumentada, GeoGebra 3D, inteligencia espacial, geometría volumétrica, educación secundaria.

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1. Introduction

Spatial intelligence is considered a fundamental cognitive component in academic and vocational education in the STEM (Science, Technology, Engineering and Mathematics) disciplines and includes the ability of individuals to perceive, manipulate and mentally represent objects, relationships and transformations in three dimensions (Uttal et al., 2013; Newcombe & Shipley, 2014; Sorby et al., 2018; Lee et al., 2020; Atit et al., 2022). This cognitive competence, which includes skills such as visualisation, mental rotation, perception of symmetry and the ability to imagine internal sections of solids, plays a key role in geometric problem solving, mathematical reasoning, critical thinking, architectural analysis, as well as in the interpretation of scientific data and the modelling of natural phenomena (Patsiomitou, 2008; Maeda & Yoon, 2013; Wang & Degol, 2017).

However, the teaching of three-dimensional geometry in secondary education faces significant challenges, as it is often based on traditional teaching methods that rely on static representation of figures and limited manipulation of learning materials. These approaches often limit students' ability to construct mental representations of geometric solids, making it difficult to understand properties, sections, spatial relationships and complex transformations (Sorby, 2009; Uttal & Cohen, 2012; Hawes et al., 2022). This problem is particularly evident in educational contexts where the curriculum includes advanced volumetric geometric concepts, but learning tools and strategies do not facilitate dynamic exploration and direct interaction with spatial entities (Nguyen et al., 2016; Hung et al., 2017).

Against this backdrop, Augmented Reality (AR) presents itself as an emerging and promising technological alternative capable of overcoming the limitations of traditional training methods by integrating three-dimensional digital elements with the real physical environment (Radu, 2014; Ibáñez & Delgado-Kloos, 2018; del Cerro & Morales, 2021; Buchner & Kerres, 2023). AR offers the possibility of creating immersive and highly interactive learning scenarios in which the learner can visualise, manipulate, rotate, decompose and reconfigure geometric objects with an immediacy and level of detail that is difficult to achieve with static images or traditional manipulative materials. In addition, this technology enhances the connection between theory and practice, promoting the construction of solid mental models, the transfer of knowledge to new contexts, meaningful learning and student motivation (Dunleavy & Dede, 2014; del Cerro & Morales, 2017).

In this area, the integration of GeoGebra 3D, a free software tool widely used in educational environments, with AR stands out as a strategy that enables the construction, manipulation and analysis of three-dimensional figures based on mathematical principles (Hohenwarter & Fuchs, 2004; del Cerro & Morales 2021; Juandi et al., 2021). Combining GeoGebra 3D with an AR environment using mobile devices enables a ubiquitous, flexible and learner-centred learning context in which the exploration of geometric properties moves beyond the boundaries of the traditional classroom (Baltaci & Yildiz, 2015; Solvang & Haglund, 2021). Students can, for example, analyse intersections between planes and solids, understand the relationship between sections and projections, and visualise rotation, translation or reflection operations in real time, increasing their conceptual understanding and their ability to cope with visuospatially intensive activities (Freina & Ott, 2015; Chang & Hwang, 2019).

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The educational relevance of this proposal is supported by previous research documenting the potential of AR and other immersive technologies in the teaching of geometry, showing improvements in academic performance, development of spatial intelligence, intrinsic motivation, cognitive engagement and attitudes towards mathematics (Crollen & Noël, 2015; Mystakidis et al., 2022). The use of standardized tests to measure spatial ability, such as the Purdue Spatial Visualisation Test: Rotations (PSVT:R), has been shown to be a psychometrically parameterised test that is valid for assessing the impact of innovative instructional interventions on students' spatial skills (Hoe et al., 2019; Gecu-Parmaksiz & Delialioğlu, 2020).

In this context, the present study adopts a quasi-experimental design with two groups: an experimental group using the GeoGebra 3D application integrated with AR, and a control group following traditional methods of teaching volumetric geometry. The analysis will compare not only performance on spatial tasks (as measured by the PSVT:R), but also academic achievement, motivation, and students' perceptions of the usefulness and effectiveness of the AR technology. The aim of the study is to provide empirical evidence of the ability of this technological integration to overcome the limitations of the conventional approach and, consequently, to promote deeper, more meaningful and transferable learning.

Research objectives:

1. Evaluate the effect of integrating AR with the GeoGebra 3D tool on improving the spatial intelligence skills of secondary school students.
2. Compare the effectiveness of the technological intervention based on AR with traditional methods of teaching volumetric geometry, taking into account indicators of academic performance, motivation and interest in the subject.

Research questions:

1. Does the implementation of GeoGebra 3D with AR lead to a significant improvement in students' spatial skills compared to traditional methods?
2. How does the intervention combining AR and GeoGebra 3D influence students' academic performance, motivation and perception towards learning three-dimensional geometry?

By answering these questions, this study aims to contribute to the literature on the use of immersive technologies in mathematics education by providing a conceptual and methodological framework that can guide both future research and the design of innovative didactic proposals. In doing so, it aims to support the incorporation of AR and GeoGebra 3D as transformative tools for teaching volumetric geometry in secondary education, thus supporting the usefulness of spatial intelligence as a key component for performance in STEM disciplines.

2. Impact of augmented reality on spatial intelligence for learning geometry

Spatial intelligence, an essential component of cognitive abilities, has been widely studied from a psychological and educational perspective due to its influence on the understanding, representation and mental manipulation of three-dimensional space

(Parong & Mayer, 2021). In the field of geometry didactics, spatial intelligence is crucial because its development has a positive impact on the formation of geometric thinking, complex problem solving and mathematical reasoning (Mix et al., 2016). Spatial intelligence encompasses a range of skills including the perception, rotation and mental representation of objects, the identification of internal sections and the understanding of topological, metric and projective relationships (Hegarty, 2010). Together, these skills facilitate the interpretation of abstract geometric concepts and their integration into students' cognitive baggage, broadening their versatility in STEM disciplines and improving their academic performance.

AR has emerged in recent years as an educational technology capable of enhancing students' spatial intelligence by providing immersive, interactive and contextualised learning environments (Wu et al., 2013). Unlike traditional methods, where geometric representations are often limited to two-dimensional diagrams printed or projected on whiteboards, AR integrates three-dimensional digital elements into the real physical environment, providing a richer, more dynamic and meaningful learning experience (del Cerro & Morales, 2017). This integration allows students to explore complex geometric solids from multiple angles, identify relationships between sections and components, manipulate objects in real time, and better understand the principles underlying spatial transformations.

In the field of mathematics education, the effectiveness of AR has been documented in several studies that highlight its positive impact on the development of students' spatial skills, motivation, attention and academic performance (Furió et al., 2013; del Cerro & Morales, 2021). This technology not only helps to overcome the barriers imposed by the static representation of content, but also enables constructivist learning, in which the student is no longer a passive recipient of information, but becomes an active agent who constructs his or her own knowledge. In this way, AR is a tool that facilitates the transition from traditional teaching focused on memorisation and reproduction of content to a pedagogy oriented towards deep understanding, experimentation and transfer of skills to new mathematical contexts (Pellas et al., 2019).

Integrating GeoGebra 3D into these immersive environments adds significant pedagogical value, as it is a versatile tool for teaching and learning geometry, providing the ability to accurately construct, manipulate and analyse three-dimensional figures, as well as explore functional relationships and metric and topological properties (Weinhandl et al., 2020). When integrated with AR, GeoGebra 3D expands its scope, allowing students to visualise spatial functions, geometric objects and their transformations directly in the physical environment. This interactive and manipulative experience contributes to a deeper and more robust understanding of geometric concepts. In addition, the availability of the application on mobile devices makes the learning process ubiquitous, flexible and highly accessible, facilitating the continuity of study beyond the classroom.

On the other hand, it is appropriate to point out the limitations of traditional teaching of volumetric geometry and how AR helps to overcome them. The representation of three-dimensional solids in printed materials or on flat screens makes it difficult to understand their internal structure, volume or sections, as the student lacks the possibility to interact directly with the object (Wassie & Zergaw, 2019). The inability to rotate, section or modify the figure limits the development of spatial intelligence, as the learner does not have enough dynamic sensory and cognitive stimuli to create and reinforce their mental images. In contrast, AR provides an immersive interaction channel that encourages active manipulation of objects, trial and error, constant experimentation and the generation of new mental representations, thus overcoming the limitations imposed by conventional methodologies.

The impact of AR on spatial intelligence for learning geometry is promising because, in combination with tools such as GeoGebra 3D, it offers not only innovative technological solutions, but also a significant didactic transformation. This transformation manifests itself in greater cognitive engagement of students, increased motivation, a deeper and more tangible understanding of three-dimensional geometric concepts, and an increase in their ability to apply this knowledge in different contexts.

While numerous studies have documented the benefits of AR and GeoGebra in mathematics education, few have investigated their specific use in improving spatial intelligence in the teaching of volumetric geometry in secondary schools. There is therefore a need for more robust empirical evidence that analyses not only the immediate impact of these technologies, but also their long-term influence on the development of students' cognitive and motivational skills.

In this sense, the present study aims to fill this gap by evaluating the implementation of GeoGebra 3D with AR in a real educational context, analysing its impact on spatial intelligence, academic performance and students' perception of their learning process. In this way, AR is positioned as a technology capable of redefining pedagogical practices in the mathematical field, contributing to the development of key spatial skills in students' education.

3. Methodology

This study is based on a methodological design aimed at evaluating the effects of the integration of AR through the GeoGebra 3D application on the development of spatial intelligence and the learning of volumetric geometry in secondary school students, specifically in the subject of Mathematics in the third year of Compulsory Secondary Education (ESO). This research adopts a quasi-experimental approach with a pretest-posttest design and non-equivalent groups, a strategy widely used in educational contexts where completely randomised assignments are not possible (Campbell & Stanley, 2015). This design allows for an effective comparison between an experimental group, using GeoGebra 3D with AR, and a control group, using traditional methods.

The sample comprised 53 students in the third year of Compulsory Secondary Education (ESO), who were randomly assigned to either an experimental or a control group. The groups were kept separate, but the same teacher delivered the sessions in both cases, thereby ensuring homogeneity in the instruction. It is noteworthy that no significant curricular adaptations were observed, thereby ensuring an initial homogeneity in the levels of skills and knowledge exhibited by the participants. Furthermore, both groups followed the same instructional schedule, curriculum content, and assessment methods, ensuring that the only variable that differed between the groups was the integration of GeoGebra 3D with augmented reality in the experimental group. The selection criteria for the sample included accessibility, student availability, and the technological resources of the school, thereby enabling the intervention to be conducted in real-world conditions representative of the educational context. This design ensured that the conditions of both groups were as equivalent as possible, with the exception of the variable that was the subject of the experiment, thus meeting the methodological requirements for validity.

The educational intervention was designed following a constructivist approach, focusing on active learning and guided exploration. The experimental group interacted with GeoGebra 3D, using mobile devices to visualise, manipulate and analyse three-dimensional geometric objects superimposed on the real environment using AR. During the sessions, activities were conducted that fostered conceptual understanding and spatial visualisation skills through multisensory experiences, aligned with previous research highlighting the effectiveness of these emerging technologies in enhancing geometric learning (Uriarte-Portillo et al., 2023; İbili et al., 2020). Figure 1 shows a photograph of the experimental group working with the GeoGebra 3D application in the classroom, reflecting the participatory nature of the experience; on the other hand, the control group continued to receive traditional instruction based on theoretical explanations and written exercises.

Figure 1. Students in the experimental group interacting with GeoGebra 3D in an AR learning environment.

Data collection was conducted using validated and standardised instruments. To assess spatial skills, the Purdue Spatial Visualization Test: Rotations (PSVT:R), a psychometrically parameterised test to measure mental rotation skills (Ernst et al., 2017), was used. This instrument was administered before and after the intervention to identify significant changes in students' spatial skills.

In addition, a Likert-type questionnaire was designed and applied, consisting of items organised into scales that specifically assessed: (1) the students' motivation towards learning with GeoGebra 3D with AR; (2) the perceived usefulness of this tool in improving their geometric learning; and (3) the perceived ease of use when interacting with the application in an AR environment. This questionnaire allowed us to obtain qualitative and quantitative data that complemented the analysis, providing a comprehensive view of the educational experience from the user's perspective. These tests, applied before and after the intervention, made it possible to accurately assess the acquisition of knowledge and skills associated with volumetric geometry, ensuring the validity and reliability of the results through Cronbach's alpha coefficient analysis (Taber, 2018). The internal reliability of the questionnaire items was assessed using Cronbach's alpha coefficient (Table 1), yielding high values in all the dimensions assessed, which supports the internal consistency of the data obtained.

Table 1.
Internal reliability coefficient (Cronbach's alpha) for the dimensions assessed.

Dimension	Cronbach's α
Alignment with curricular standards	0.872
Effectiveness of AR in problem-solving	0.789
Ease of interaction with AR tools	0.845
Engagement and interest stimulated by AR	0.911
Perceived reliability of the app	0.896

In order to measure academic performance in volumetric geometry, specific written tests were designed in line with the official Mathematics curriculum for the third year of ESO in Spain, established in Royal Decree 217/2022 (Ministry of Education and Vocational Training, 2022). The curriculum emphasises the development of skills related to the understanding and analysis of three-dimensional shapes, as well as the resolution of problems involving the calculation of areas, volumes and spatial relationships. During the four-week intervention, the mathematics contents addressed were aligned with these criteria, focusing on the following topics:

1. Recognition and representation of geometric solids: identification of prismas, pyramids, cylinders, cones and spheres, emphasising their characteristics and properties.
2. Area and volume calculation: application of formulae to determine lateral and total areas, as well as the volumes of geometric solids.
3. Section analysis: interpretation and visualisation of plane cuts in solids, exploring how these affect shape and volume.
4. Geometric transformations: visualisation and manipulation of solids using rotations, translations, and reflections to develop spatial skills.

Within the volumetric geometry block, the assessment criteria include the identification and representation of three-dimensional figures, the use of formulas for the calculation of areas and volumes of geometric solids, and the ability to solve problems combining these concepts. The tests designed incorporated items addressing the following specific contents: (1) recognition and representation of geometric solids such as prisms, pyramids, cylinders, cones and spheres; (2) calculation of lateral and total areas of solids, employing specific formulae according to the figure; (3) determination of volumes of solids by using standard formulae, including situations that required decomposition into simpler solids; (4) solving problems involving comparisons between areas and volumes, applying logical and geometric reasoning; and (5) interpreting plane sections of solids, analysing how cuts affect the shape and volume of the figures.

In addition, the activities carried out in the experimental group included the use of GeoGebra 3D with AR for surface detection and interactive representation of three-dimensional figures, as illustrated in Figure 2, which shows the GeoGebra 3D interface detecting the surface (Figure 2.a), and performing a geometry introduction and representation activity with different polyhedra (Figure 2.b).

Figure 2. GeoGebra 3D interface: (a) Surface detection and (b) Activity introducing and interactively representing three-dimensional polyhedra in an augmented environment.

The analysis of the data collected followed a statistical approach where descriptive analyses were performed to calculate means, standard deviations and ranges, characterising the behaviour of the variables before and after the intervention. Subsequently, inferential analyses were used, such as Student's t-tests for independent samples and analysis of covariance (ANCOVA), with a confidence level of 95% ($p < 0.05$). This approach allowed us to determine the statistical significance of the differences observed between the experimental and control groups. Measures of effect, such as Cohen's coefficient (Cohen, 1988), were also calculated to assess the magnitude of the impact of the intervention.

From an ethical perspective, compliance with the principles of informed consent, anonymity and confidentiality of data was ensured. Participants and their families were informed of the research objectives and were allowed to withdraw at any time without negative repercussions. To ensure the feasibility of the study, initial training on the use of GeoGebra 3D and technical assistance was provided during the sessions, ensuring that students were able to take full advantage of the capabilities of the technology implemented in the classroom experience.

4. Results

This section presents the main findings of the study, emphasising the impact of AR and the GeoGebra 3D tool on the development of learning in volumetric geometry and spatial skills. The results are structured considering the experimental design, the applied statistical analysis and the observed implications on student motivation and perception. The results are presented below with updated tables including Cohen's *df*, *p* and α values for all comparisons. The data have been contextualised and described in a way that is consistent with the theoretical and methodological framework previously developed, providing clarity in the interpretation of the results. In addition, we include inferential and descriptive analyses supported by references, which strengthen the validity of the findings and allow us to draw meaningful conclusions about the impact of the intervention.

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4.1. Procedure

The development of the research was structured in three main stages. In the first phase, a pre-test was carried out that included the application of the PSVT:R and an initial written test, with the purpose of establishing a baseline level of spatial skills and geometric knowledge in both participating groups, which allowed us to guarantee the initial homogeneity of the samples and establish a baseline to evaluate the impact of the intervention. Figure 3 shows an example of a question used in the PSVT:R, where students' ability to correctly identify how a three-dimensional object is rotated in space is assessed.

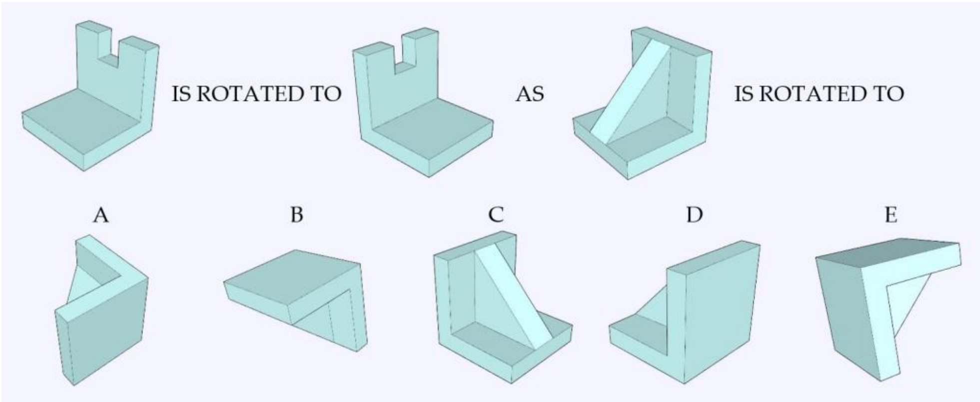

Figure 3. PSVT: R example question.

The second stage consisted of the implementation of the intervention, which lasted approximately four weeks, adjusting to the students' time availability and pace of progress. During this period, the experimental group used the GeoGebra 3D tool in an AR environment, where students participated in hands-on activities focused on the manipulation of geometric solids, the calculation of volumes and areas, and the dynamic exploration of geometric properties (Figure 4). These activities are designed to foster interactive learning, allowing students to relate abstract concepts to concrete, visual experiences.

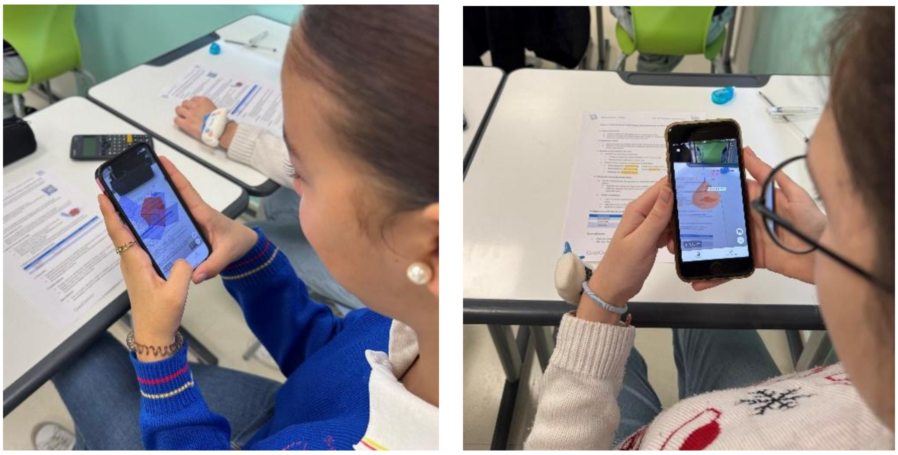

Figure 4. Students in the experimental group working in the classroom during the experience with GeoGebra 3D and AR.

Finally, in the third stage, a post-test was carried out which again included the PSVT:R, the final written test and a perception questionnaire designed to assess students' motivation and experience with the technological tools used. In order to foster student autonomy and facilitate access to the content, worksheets enriched with AR elements in the form of activity cards were designed as open educational resources (OER) (Morales et al., 2024). These worksheets integrated QR codes that linked directly to the GeoGebra AR application, allowing students to interact with augmented activities in an intuitive and accessible way, an example of these worksheets is shown in Figure 5.

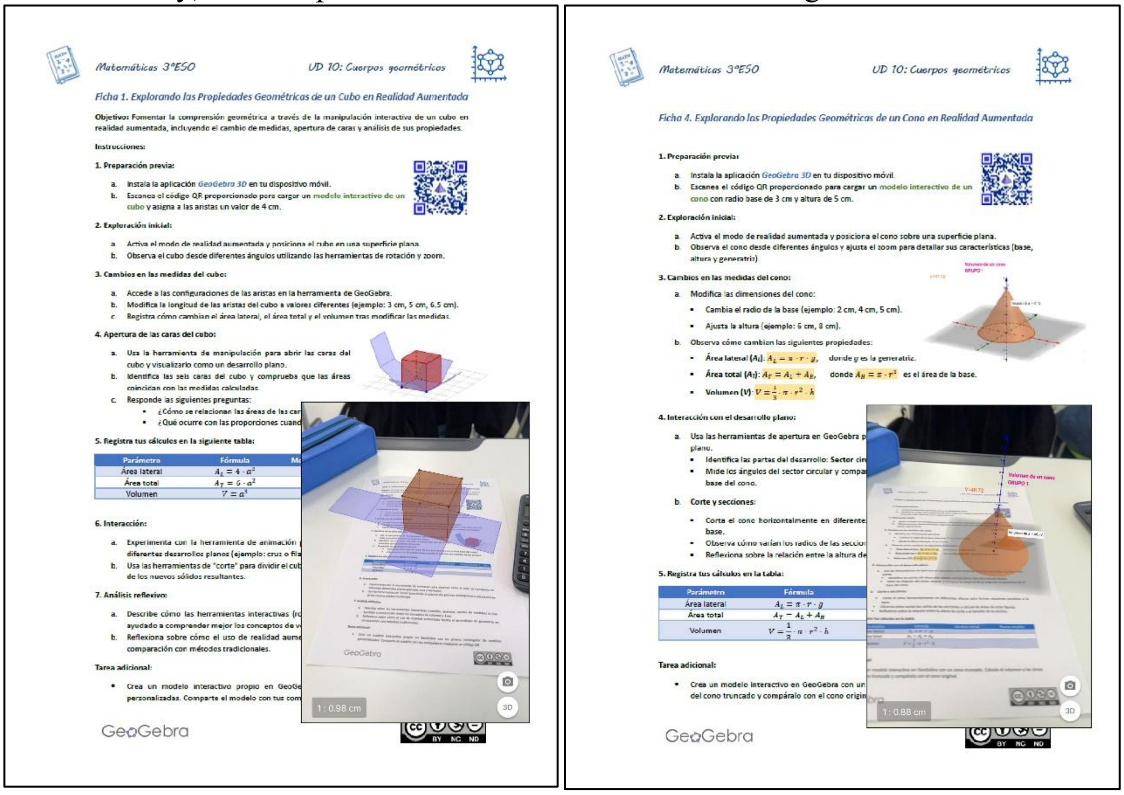

Figure 5. Example of worksheets with AR content, accompanied by an illustrative screenshot of the augmented content in the application.

4.2. Data analysis

Data analysis was performed using descriptive and inferential statistical techniques, Student's t-tests for independent samples with a significance level of 5% ($p < 0.05$) were carried out to determine significant differences between the experimental and control groups. Effect sizes were also calculated using Cohen's α coefficient to assess the magnitude of the differences found. Additionally, the use of analysis of covariance (ANCOVA) was considered to adjust for possible initial variations and provide a more accurate analysis.

The results are presented in a structured way according to the main variables evaluated: spatial skills, academic performance in volumetric geometry and student perception of the intervention, highlighting both the values obtained and their interpretation.

The PSVT:R was used to measure spatial skills before and after the intervention, the results of which are summarised in Table 2, showing significant improvements in the experimental group after the intervention. While differences in the pretest were not significant ($df = 0.24$, $p = 0.81$), the posttest revealed a significant improvement in favour of the experimental group ($df = 2.31$, $p = 0.025$) with a moderate effect size (Cohen's α = 0.60). This finding reinforces the idea that the use of GeoGebra 3D in an AR environment facilitates the development of visualisation and mental rotation skills, fundamental skills for geometric learning.

Table 2.
Descriptive statistics of PSVT:R and results of between-group t-tests.

Test	Group	n	M	DE	Min.	Max.	*df*	*p*	Cohen's α
Pretest	Exp	27	14.2	3.4	9	20	0.24	0.81	0.05
Pretest	Ctrl	26	14	3.1	8	19	0.24	0.81	0.05
Posttest	Exp	27	17.5	3.2	12	23	2.31	0.025	0.6
Posttest	Ctrl	26	15	3.2	9	20	2.31	0.025	0.6

Note: In the pre-test, no significant differences were observed between Exp and Ctrl. In the post-test, the Exp group showed a significant improvement compared to Ctrl. $p < 0.05$.

On the other hand, academic performance in volumetric geometry, assessed by a written test, also showed a significant improvement in the experimental group after the intervention. As detailed in Table 3, while in the pretest no significant differences were found ($df = 0.25$, $p = 0.80$), in the posttest the scores of the experimental group were significantly higher ($df = 2.04$, $p = 0.047$). Although the effect size was moderate (Cohen's α = 0.40), these results underline the effectiveness of AR-based learning in promoting deeper understanding of complex geometric concepts.

Table 3.
Descriptive statistics of academic performance and t-test results between groups.

Test	Group	n	M	DE	Min.	Max.	*df*	*p*	Cohen's α
Pretest	Exp	27	5.5	1.2	3	8	0.25	0.8	0.05
Pretest	Ctrl	26	5.4	1.3	3	8	0.25	0.8	0.05
Posttest	Exp	27	6.7	1.1	4	9	2.04	0.047	0.4
Posttest	Ctrl	26	5.9	1.2	4	8	2.04	0.047	0.4

Note. In the post-test, significant differences were observed in favour of the Exp group, although the effect is moderate $p < 0.05$.

Finally, students' perception of the intervention was assessed through a questionnaire that included dimensions such as perceived usefulness, motivation and ease of use. The results, presented in Table 4, highlight that perceived motivation was significantly higher in the experimental group ($df = 2.10$, $p = 0.04$, Cohen's α = 0.6). Although perceived usefulness and perceived ease of use did not differ significantly, the data indicate a positive trend towards experience with AR tools.

Table 4.
Descriptive statistics of academic performance and t-test results between groups.

Dimension	Group	n	M	DE	*df*	*p*	Cohen's α
Perceived usefulness	Exp	27	4.5	0.8	1.48	0.145	0.4
(1-3)	Ctrl	26	4.1	0.9	1.48	0.145	0.4
Motivation	Exp	27	4.8	0.9	2.1	0.04	0.6
(4-6)	Ctrl	26	4	1	2.1	0.04	0.6
Ease of use	Exp	27	4.6	0.7	1.78	0.08	0.5
(7-10)	Ctrl	26	4.2	0.8	1.78	0.08	0.5

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Note. The motivation dimension shows significant differences in favour of the Exp group $p < 0.05$.

Overall, the results obtained show that the AR-based intervention and the use of GeoGebra 3D not only contribute significantly to the development of spatial skills and the increase in academic performance, but also enhance students' motivation towards learning. These findings highlight the importance of integrating emerging technologies into educational environments, promoting deeper, more interactive and enriching learning that responds to the needs of contemporary education.

5. Discussion

The findings substantiate the efficacy of incorporating interactive technological tools into the pedagogy of volumetric geometry, thereby corroborating extant research that has previously underscored the potential of AR. Specifically, the substantial enhancement in spatial aptitude exhibited by the experimental group, as gauged by the PSVT:R, aligns with the observations reported by İbili et al. (2020). These researchers contend that AR fosters mental rotation and spatial visualisation through intuitive interfaces. The statistically significant difference between groups (df = 2.31, p = 0.025, Cohen's alpha = 0.6) indicates a moderate effect due to interactive manipulation of 3D objects in a realistic environment.

The capacity of GeoGebra 3D to enable direct exploration, characterised by rotational, zoological and perspective manipulation, has been demonstrated to facilitate the establishment of enduring and transferable cognitive frameworks. This finding aligns with the assertions made by Uriarte-Portillo et al. (2023), who contend that AR integrates abstract and concrete elements, thereby promoting comprehension within immersive three-dimensional environments. The enhancement in academic performance related to volumetric geometry (df = 2.04, p = 0.047, Cohen's α = 0.4) further substantiates the notion that direct interaction with three-dimensional models fosters conceptual understanding, corroborating the observations of Solvang and Haglund (2021), who have demonstrated the efficacy of dynamic exploration in the assimilation of concepts pertaining to volumes and sections of solids. These results align with the findings of studies such as Bautista et al. (2024), which emphasise the potential of augmented reality to influence the spatial arrangement of content and reduce cognitive load, thereby facilitating the comprehension of complex concepts in geometry and other STEM disciplines.

Moreover, the incorporation of AR components fostered discernible connections between theoretical concepts and their practical applications in geometry, a phenomenon emphasised by Juandi et al. (2021) in their analysis of GeoGebra 3D. The projection of mathematical functions or objects in the physical environment was found to be a significant factor in increasing students' motivation and positive perceptions of learning, as evidenced by the substantial difference in motivation observed in the experimental group (df = 2.1, p = 0.04, Cohen's α = 0.6). These observations corroborate the findings of Cerro and Morales (2021), who highlight the autonomous and collaborative approach offered by OER enriched with AR.

Although there were no statistically significant differences in perceptions of usefulness and ease of use, qualitative comments indicated a favourable trend towards the tool, despite specific challenges such as compatibility with older devices. This finding is in accordance with the conclusions of Pellas et al. (2019), who argued that external factors, including technological infrastructure and technical support, influence the adoption of AR solutions.

Notwithstanding the encouraging results, limitations are identified that engender a degree of caution in relation to the generalisation of the findings. The relatively brief intervention period (four weeks) hinders the ability to ascertain the sustained effects over time, and the constrained sample size limits the potential for extrapolating conclusions to other educational contexts. Furthermore, teacher training and the availability of compatible technological resources are identified as critical challenges for the widespread implementation of AR in the classroom.

The results, when considered as a whole, highlight the potential of AR in the context of STEM education. The integration of visual, tactile and conceptual stimuli has been demonstrated to enhance student engagement, strengthen motivation and promote deeper understanding of abstract concepts. These findings serve to confirm GeoGebra 3D as an accessible and versatile tool in science and mathematics education.

6. Conclusions

The findings indicate that AR, particularly when utilising the GeoGebra 3D software, has the potential to significantly impact the pedagogy and learning of volumetric geometry within the secondary education sector. By facilitating interactive engagement with three-dimensional models in real time, students have been shown to develop spatial skills and demonstrate significant improvements in academic performance, as evidenced by increased scores on PSVT:R and achievement tests.

The direct manipulation of three-dimensional objects and the possibility of projecting complex concepts in a physical environment have been shown to facilitate a deeper understanding of metric and topological properties. This approach has been demonstrated to encourage autonomous and critical learning. This advancement in the cultivation of STEM competencies is further bolstered by heightened student motivation, indicating that technological innovation not only fosters conceptual understanding but also catalyses engagement and enthusiasm for mathematics.

Nevertheless, the study is not without its limitations, which must be considered when interpreting the findings. The quasi-experimental design, the sample size and the short duration of the intervention may influence the external validity and the evaluation of long-term effects. Furthermore, the dependence on technological infrastructures and the need for teacher training point to organisational challenges that educational institutions need to address to ensure the sustainability of these innovations.

The evidence presented substantial supports the incorporation of AR as an effective pedagogical resource for teaching volumetric geometry, by providing dynamic and tangible learning experiences that reinforce conceptual understanding and motivation. Future studies could extend the intervention period, increase the sample size and consider different educational populations to assess the persistence of the effects and enhance the implementation of immersive technologies in the classroom.

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