Revista **Educatio Siglo XXI**

ISSN edición impresa: 1699-2105. ISSN edición web (http://revistas.um.es/educatio): 1989-466X

Una nueva herramienta para evaluar la comprensión del cálculo confiando en la comprensión relacional e instrumental[1]

A New Tool for Assessing the Understanding of Calculus by Relying on Relational and Instrumental Understanding

HOJJAT MOOSAVI
Islamic Azad University, Irán
hojjat_moosavi60@yahoo.com
https://orcid.org/0000-0003-3385-710X
ABOLFAZL TEHRANIAN[2]
Islamic Azad University, Irán
Tehranian1340@yahoo.com
https://orcid.org/0000-0002-5811-1139
HOSSEIN DOOSTIE
Islamic Azad University, Irán
doostih@gmail.com
https://orcid.org/0000-0002-4301-8070
AHMAD SHAHVARANI
Islamic Azad University, Irán
shahvarani.1393@gmail.com
https://orcid.org/0000-0002-4602-5743

Resumen:
Los cambios en los libros de matemáticas de la escuela secundaria en los últimos años han llevado a algunos cambios en

Abstract:
The changes in high school mathematics books in recent years have led to some changes in the concept of calculus, and

1 **Como referenciar este artículo (How to reference this article):**
Moosavi, H., Tehranian, A., Doostie, H., y Shahvarani, A. (2022). Una nueva herramienta para evaluar la comprensión del cálculo confiando en la comprensión relacional e instrumental. *Educatio Siglo XXI*, *40*(1), 55–78. https://doi.org/10.6018/educatio.432431

2 **Dirección para correspondencia (Correspondence address):**
Abolfazl Tehranian. Department of Mathematics, Science and Research Branch, Islamic Azad University, Tehran (Iran).

el concepto de cálculo, y la mayoría de los estudiantes solían trabajar duro en la memorización en lugar de la comprensión. En los primeros años después de ingresar a la universidad, serán desafiados por los mismos conceptos y su aplicación en otras ciencias. La revisión de los estudios anteriores ha demostrado que hasta ahora no se ha desarrollado ningún instrumento de investigación para abordar tales desafíos. Mediante metodología descriptiva, los investigadores del estudio desarrollaron y estandarizaron un instrumento. La población estaba compuesta por todos los estudiantes universitarios de primer año de los campos de ingeniería y ciencias básicas de la Universidad de Teherán. Los investigadores consideraron algunos factores y desarrollaron un instrumento y finalmente lo validaron en base a esos componentes. Dado que no había herramientas disponibles para medir la comprensión relacional e instrumental de la derivada, se diseñaron algunas preguntas sobre el concepto de definición de derivada y se agregaron al final del cuestionario. Como resultado, luego de las investigaciones realizadas a través del análisis factorial exploratorio, se desarrolló e introdujo un instrumento de investigación para medir la comprensión del concepto de derivada y su comprensión instrumental y relacional con cinco factores. Entonces parece que el cuestionario respetado constaba de varias secciones de preguntas textuales y computacionales que pueden estimar la comprensión de los estudiantes de la derivada.

Palabras clave:
Cálculo; derivadas; comprensión matemática; comprensión instrumental; comprensión relacional.

majority of learners used to work hard on memorizing rather than understanding. In the early years after entering university, they will be challenged by the same concepts and their application in other sciences. Review of the previous studies have shown that no research instrument has been developed to address such challenges till now. Through descriptive methodology, the researchers of the study developed and standardized an instrument. The population was made up of all first-year undergraduate students of engineering and basic sciences fields from of University in Tehran. The researchers considered some factors and developed an instrument and finally validated it based on those components. Since there were no available tools to measure the derivative's relational and instrumental understanding, some questions regarding the concept of derivative definition were designed and added to the end of the questionnaire. As a result, after the investigations conducted through exploratory factor analysis, a research instrument was developed and introduced to measure the understanding of the concept of derivative and its instrumental and relational understanding with five factors. Then it seems that respected questionnaire consisted of several sections of textual and computational questions can estimate the students' understanding of the derivative.

Key words:
Calculus; derivative; mathematical understanding; instrumental understanding; relational understanding.

Résumé:
Les modifications apportées ces dernières années aux manuels de mathématiques de l'enseignement secondaire ont entraîné certains changements dans le concept de calcul, et la plupart des élèves avaient l'habitude de travailler dur sur la mémorisation plutôt que sur la compréhension. Au cours des premières années suivant leur entrée à l'université, ils seront confrontés aux mêmes concepts et à leur application dans d'autres sciences. L'examen des études antérieures a montré que, jusqu'à présent, aucun instrument

de recherche n'a été développé pour relever ces défis. En utilisant une méthodologie descriptive, les chercheurs de l'étude ont développé et standardisé un instrument. La population était composée de tous les étudiants de première année de licence en ingénierie et en sciences fondamentales de l'université de Téhéran. Les chercheurs ont pris en compte certains facteurs et ont développé un instrument et l'ont finalement validé sur la base de ces composantes. Comme il n'y avait pas d'outils disponibles pour mesurer la compréhension relationnelle et instrumentale de la notion de dérivé, quelques questions sur la définition du concept de dérivé ont été conçues et ajoutées à la fin du questionnaire. En conséquence, après des investigations par le biais d'une analyse factorielle exploratoire, un instrument de recherche a été développé et introduit pour mesurer la compréhension du concept de dérivé et sa compréhension instrumentale et relationnelle avec cinq facteurs. Il semble alors que le questionnaire respecté comprenait plusieurs sections de questions textuelles et calculatoires qui peuvent estimer la compréhension de la dérivée par les élèves.

Mots clés:
Calculus; dérivé; compréhension mathématique; compréhension instrumentale; compréhension relationnelle.

Fecha de recepción: 14-06-2020
Fecha de aceptación: 16-09-2020

Introduction

One of the important branches in the field of mathematics is calculus that has a wide range of applications in all areas such as business, computer science, and information system. Calculus is regarded as the preliminary course to the higher-level mathematics required by mathematicians, doctors, statisticians, engineers, and scientists for many fields of study regard to education systems in countries. Many students consider calculus as a key factor that helps them to decide whether they seek to have a degree in one of the related fields or shift to a career path that needs a lower level of math. If the students plan to access enough knowledge in one of mathematical fields, it is crucial for them to gain a great understanding of calculus (calculus includes of basic knowledge of mathematics in secondary such as functions, limit, derivative, Integral, so on). Thus, observing the fact that many students fail the introductory calculus is discouraging. Sahin et al. (2015) in the study of flipping a college calculus course argued that due to the complex nature of calculus which includes abstract ideas and concepts and the method of its teaching to students, this mathematical branch is considered as the major source of failure in secondary school level. Every year a large percentage of students enter colleague without sufficient preparation for the difficult level of college work (Tierney & Garcia, 2008). Calculus is consisted of the two parts of differential and integral. Derivative which is the central idea of calculus, is the first part of the differential calculus which represents the function's instantaneous rate of change. Derivative, like integral, is derived from a geometrical problem, which is the finding of a tangent line at a specific point on a curve. The concept of derivative was not developed until the early 17th century, when Pierre de Fermat, the French mathematician, set out to determine the extrema of some functions. Fermat found that the tangent lines must be horizontal at the maximum or minimum points of a curve. At first glance, it seemed that there was no relationship between the problem of calculating the area under a graph and the issue of determining the tangent line on a curve at a given point, but the first person who found that these two apparently distant concepts were closely related, was Isac Barrow, the teacher of Isaac Newton (Smith, 2015). Derivative has been defined in several ways by different researchers. According to the definition proposed by Balcı (2012) based on limit, derivative is considered as the limit of the ratio while change of independent variable is close to zero, in order to increase the independent variable of the function. Karadeniz (2003) addressed the geometric definition of this concept, according to which, a function's derivative is in fact the tangent of the angle between x-axes (which is also the slope of the tangent) and the specified point of the function. Reviewing the literature reveals that there are a wide range of studies that focus on the different challenges regarding the subject of derivatives (e.g. Hähkiöniemi, 2005; Gür & Barak, 2007; Aksoy, 2007; Özkan & Ünal, 2009; and Bingölbali, 2010). Research in this domain has shown that students do not have a proper understanding of the concept of derivative and have many problems with it (Roundy et al., 2015; Pino-Fan et al. 2018). Many high school students in their final years of high school and freshmen have fundamental problems understanding the derivative. Therefore, it is necessary to provide educational opportunities to give meaning and objective meaning to the abstract concept of derivation (Sánchez-Matamoros-Garcia et al., 2008). Although the problem with understanding the concept of derivative in calculus is not something new, it is still main challenge in mathematics education at the university level (calculus is the base of solving problems of economic, management, basic sciences, educational and medical fields) and it is considered a constant concern for higher education institutions, because

 https://doi.org/10.6018/educatio.432431

it leads to students' low scores and a high number of students that fail their exams and drop out of school in the course of calculus (Bressoud et al., 2015). Derivative understanding is relative among college students, and in their view, derivation is a complex subject. Therefore, new educational interventions such as representation, schema, etc. should be done to teach such an abstract concept (Fuentealba et al., 2019). Bouguerra (2019) believed that misunderstanding the derivative leads to several misunderstandings in derivative concepts. Mathematicians, economists, physicists, engineers, and other experts often use it in order to solve the problems in real life (Laridon et al., 2007). For instance, it is applicable to the researches on wireless and electric lighting, machinery of all kinds, optics, and thermodynamics (Rohde et al., 2012). In business and economics, differential calculus is useful to resolve problems associated with obtaining minimum cost or maximum profit and other related values (Berresford & Rockett, 2015). Students' conceptual understanding of the derivative can help teach and learn the basic concepts of differential and integral calculus (Lee, 2018). The texts in high school mathematics books (for example; derivative) in secondary differ of the texts of concept calculus in theoretical fields of study including humanities, mathematics, and sciences in the university in terms of problems that propose in that the field of study. Then when students mostly focus on memorizing and rote learning rather than understanding in order to pass the university entrance examination, it seems that in the early years after entering university they will be challenged by the same concepts and their application in other sciences.

Therefore, the researchers of this study seek to identify the challenges of understanding one of the calculus topics from different angles in relation to derivative concept which is the basis of many other concepts in other sciences. Reviewing the related literature indicates that several challenges have been remained unexamined from the perspective of teaching and understanding derivatives, and no tools have been developed to address such challenges. Based on this issue and by relying on some previously conducted studies, the researchers of this study developed and standardized an instrument to address these challenges, and finally considered a model of the relationship between the challenges ahead in understanding and teaching the concept of derivative. One of the necessities of studying such a subject is to investigate and find the roots of the existent challenges in students' mathematical understanding in the basic and main courses of university. Since Skemp was the only one who had worked upon and studied both instrumental and relational aspects of mathematical understanding, and no other similar and objective study has been conducted in this field, we consider it necessary to investigate these two aspects of mathematical understanding regarding the concept of derivative, by university students.

Theoretical Foundations of The Derivative Understanding

Cognitively speaking, understanding must be related to a range of knowledge and appropriate content. For any content, understanding as a mental status of cognition, might be conceived as a mental process, or an ability. Content understanding may include 'knowing one thing ...', 'knowing how' and 'knowing why' regarding that specific thing (Woods & Barrow, 1975). In mathematics, 'knowing one thing' refers to the knowledge of facts, concepts, and principles (Van Engen, 1953). 'Knowing how' refers to skills that include mathematical operations and procedures that are performed according to a set of rules, instructions, or algorithms. This issue is closely related to the common concept that if you understand the rule, you must know how to act based on it. Skemp (1978) defined 'instrumental understanding' as knowing what needs to be done without any reason. He noted that this type of understanding is probably a rote understanding, which is divided into several rules, including direct application and is effective for a short time period. The advantage of instrumental understanding is that the learner can quickly get the right answer. Teaching in regular classrooms tends to emphasize this kind of understanding. 'Knowing why', justifies 'knowing one thing' and 'knowing how' that thing works. Skemp (1976) considered 'knowing what to do and why' as a 'relational understanding' and considered 'conforming to its accepted forms of presentation' as a 'logical understanding'. Failure to address any discrepancies in what should be achieved by 'learning to understand' can lead to learning problems (Skemp, 1976). The teachers' failure in teaching mathematics may basically originate from the fact that teachers do not have enough consideration that mathematical mastery involves many intertwined and interdependent competence, such as problem posing and solving, reasoning, representing, modelling, handling symbols and formalism, exploiting tools and aids, and com-

 https://doi.org/10.6018/educatio.432431

municating (Jaworski, 2015). It should be also kept in mind that different interlinked motives and activities are related to the phylogeneses of mathematical knowledge, like calculate, apply, construct, evaluate, play, find, argue, and order (Haapasalo & Zimmermann, 2015). Skemp (1976) believes that simply understanding how to perform mathematical procedures is not sufficient for learners, they also must know the reason behind performing a specific task. In his research, the learners should know how to perform differentiate exploiting first principles:

$$\frac{dy}{dx} \vee f'(x) \vee y' = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

And, they should know rules of differentiation to obtain the derivatives of given functions. The knowledge of procedures is called procedural understanding/fluency. It is the knowledge of using procedures in appropriate ways and occasions, and the knowledge of skills to perform those procedures in a flexible, accurate, and efficient way (Kilpatrick et al., 2001; Watson & Sullivan, 2008). Procedural knowledge is constructed on conceptual understanding (NCTM, 2014). Learners' understanding of derivatives concerning the concept of derivatives was examined considering Skemp's (1976) theory on developing the concept of mathematical understanding: relational understanding (i.e., understanding both what to do and why) and instrumental understanding (i.e., knowing rules without reasons). Skemp (1976) referred to relational understanding as knowing what to do and the reason it must be done. Therefore, it can be understood that relational understanding is the understanding of learners in knowing which procedures to be performed and the logic behind each step in the procedure that will be conducted. A rational understanding of the concept of derivative requires making sense of specific relations among derivative, the concept of limit, the slope of tangent, the rate of change, and the slope of tangent. It involves not only knowing, but also being capable to describe the function of these concepts regarding its meaning and the reason and the way it is related to derivative. However, Star & Stylianides (2013) believed that learning to promote rational understanding is really demanding since it must develop a structure of concepts from a principle that generates lots of unlimited plans to build a concept. Fostering rational understanding takes a lot of time, this type of understanding can develop including building a correct understanding; training students to notice the problem; developing skills in applying mathematical principles; creating inductive abilities. In contrast, the instrumental understanding of the derivative concept refers to the knowledge without making sense of the meaning of these concepts and the way they are interrelated in the context of derivative. Many studies have been conducted on the notion of relational and instrumental types of understanding so far. One of these studies that was conducted by Bahar et al. (2012) indicated that considering the issue of learning derivatives, while the low achieving participants represented to have instrumental understanding, the medium and high achieving participants showed to have relational understanding of this concept. Some of the studies conducted in this area are briefly reviewed as follows: Sahin et al. (2015) conducted a study on the students' understanding and awareness regarding the relationships and connections among the main ideas which shape the derivative concept. For this purpose, using Skemp's dichotomy of instrumental and relational understanding, and modeling tasks, three second-year university students were undergone this investigation. According to the results of their study it was found that the students participated tin their study relatively had instrumental understanding of the derivative concept. Their findings underlined that although one of the main ideas is overlooked, the derivative concept might not be fully understood in terms of the relational understanding, as a result of the classification of these main ideas in learners' conceptual systems. Denbel (2015) showed that derivation as a mathematical subject is one of the central concepts in learning university level calculus. Students mostly have problems in learning the concept of derivative, which is mostly due to the students' lack of conceptual understanding and the fact that they mostly concentrate on procedural aspects rather than conceptual ones. He found that students have serious problems and many difficulties in conceptual understanding of derivatives. The responses provided by the students showed that their difficulties and problems in conceptual understanding of derivation is mostly due to paying more attention to the symbolic aspect compered to embodied aspect, inability to logically connect these aspects to each other, and lack of enough strength in working with generalized forms of questions. In a study by Desfitri (2016), teachers' level of understanding on the concepts of derivative and limits, and the way they teach and transfer these mathematical

 https://doi.org/10.6018/educatio.432431

subjects in their classes were analyzed. Using the Structure of the Observed Learning Outcome (SOLO) Taxonomy, the researcher investigated the teachers' level of complexity on this topic. It was found that majority of teachers were placed on the third level of this taxonomy. It was also found that half of the participants faced difficulties in their teaching practice considering the concept of derivative because of their limitations and lack of mastery on this subject matter.

According to Ayebo et al. (2017), the related studies have shown that the large number of students who did not pass the university calculus have led to the decrease in the number of students entering the majors and fields which are closely related to mathematics. Ayebo et al. (2017) used an eclectic method of study to investigate the relevance of the high school level mathematical preparation, extra trainings in university, and the success in preparatory calculus. Their findings indicated that if the students be well-prepared in high school for pre-calculus, it will indeed help them to make a big progress in their future success in university-level calculus and in their following courses of advanced mathematics. Widada et al. (2018) conducted a study to describe the interlevel students' ability to understand and comprehend the concept of derivative functions while engaged in learning the ethnomathematics. The study was conducted in a partaking way in the routine process of learning using ethnomathematics method. Based on the findings of this study, the students could summarize the properties of the function's process or intervals in that area, so the object is shaped like the illustrated function graph. Maharaj & Ntuli (2018) investigated the university students' ability and skill to appropriately use the rules for the functions' derivatives with the various structures which they faced in their studies. The results of this study showed that the participants had difficulty and they faced problems in recognizing that there was a need for multiple rules considering the derivatives, in order to differentiate specific types of functions illustrated in a symbolic form. Fuentealba et al. (2019) conducted a study in which he identified and described the development levels of derivative schema through the use of the framework suggested by the APOS theory, which worked based on the formation of 27 variables that permitted the dissection of the resolution procedures from the questionnaire into separate elements. Using a cluster analysis, the students' which belonged to different levels of derivative schema development of were identified and determined. Then, a statistical frequency analysis and implicative analysis were conducted with the 27 variables, which permitted to illustrate the determined development levels.

Research Method and Instrument

This research work is a descriptive study with methodological approach from the instrumentalization and standardization types. In this research methodology, based on theoretical foundations and by relying on the main aims and sub-aims of the study according to the questionnaire obtained from Yoong's (1984) study on mathematical understanding, the researchers of this study considered some components and developed and finally validated a research instrument based on those components. Since there was no available tool to measure relational and instrumental types of understanding derivatives, at the end of the mathematics comprehension questionnaire, 10 items were designed and added from the concept of derivative definition based on the two books by Hoffmann & Bradley (2010) and Thomas et al. (1988). In the first step, the content validity ratio index with CVI and CVR indices that were respectively attributed to Waltz & Bausell and Lawshe was examined. The obtained results after two sessions of questionnaire distribution among eight experts in the fields of pure mathematics and mathematics education are as follows:

Three questions had to be removed from the set of derivative questions for not reaching the standard level related to the above-mentioned indexes, and also the sentences of some questions from 1 to 50 were changed based on the content validity index and finally 57 items of the questionnaire were approved. In the second step, after examining its content validity and reliability that was determined and approved for each factor in the section of exploratory factor analysis with Cronbach's alpha values, the survey form was compiled in two parts; the first part included demographic information such as gender, age, and field of study, and the second part, consisted of 57 items with five Likert scales that main content of 50 explained in Table 4, and questions 51 to 57 explained in Appendix. It should be noted that the 5-point Likert scale for the 50 items of the questionnaire included Completely Disagree, Disagree, Neutral, Agree, and Completely Agree. To measure relational and instrumental understanding of derivatives in the seven relevant items, Likert scales

 https://doi.org/10.6018/educatio.432431

were designed to carefully examine relational and instrumental understanding based on Skemp's principles. These scales, the first two of which show instrumental understanding with low and high intensity, and the last two of which show the relational understanding with low and high intensity, were as follows:

- Only used basic derivative definitions and formulas and had sufficient mastery.
- Used the basic definitions and formulas of the derivative incompletely and did not have sufficient mastery.
- None of them / Unanswered
- Has a relatively sufficient mastery in establishing the relationship between the derivative's basic definitions and formulas and problem solving, but cannot establish complete and accurate adaptability and generalization between the derivative's basic definitions and formulas and problem solving.
- Has a sufficient mastery in establishing the relationship between the derivative's basic definitions and formulas and problem solving, and can establish complete and accurate adaptability and generalization between the derivative's basic definitions and formulas, and problem solving.

It should be noted that the researcher's choice was also reviewed and approved by several experts in the field of mathematics education.

Participants

The statistical population consisted of all first-year undergraduate students of engineering and basic sciences fields from four branches of Islamic Azad University in Tehran including Science and Research branch, South Tehran branch, Shahr-e-Rey Branch, and Tehran North Branch. Using purposeful sampling technique, 162 male and female students (whose math scores were above average) were considered as the sample of the study during the academic year 2019-2020. All the participant selected for the study had passed the course of calculus 1. 60% of the students were female and 40% of them were male. Their average age was about 19 years.

Findings

In this study, 57 items were designed to measure the factor variables. To investigate whether the data obtained from this study are suitable for exploratory factor analysis, we used two statistics.

Table 1
KMO and Bartlett's Test

0.88	Kaiser-Meyer-Olkin Measure	
3199.03	Approx. Chi-Square	Bartlett's Test of Sphericity
0.000	*p*	

Nota. *$p < .05$

In Table 1, since the KMO index is 0.88, and because this value is greater than 0.7 and close to one, the number of samples is sufficient for factor analysis. The P-value of Bartlett's test is less than 0.05, which shows that factor analysis is suitable for identifying the structure of the factor model. The initial commonalities are the total value of variance of a variable that the set of factors can explain it.

Table 2
Communalities

Items	Extraction	Items	Extraction	Items	Extraction	Items	Extraction
Q1	0.73	Q17	0.80	Q33	0.68	Q49	0.82
Q2	0.73	Q18	0.71	Q34	0.59	Q50	0.83
Q3	0.58	Q19	0.71	Q35	0.53	Q51	0.58
Q4	0.70	Q20	0.63	Q36	0.67	Q52	0.59
Q5	0.54	Q21	0.81	Q37	0.64	Q53	0.64
Q6	0.61	Q22	0.83	Q38	0.53	Q54	0.55
Q7	0.48	Q23	0.82	Q39	0.62	Q55	0.67
Q8	0.48	Q24	0.81	Q40	0.81	Q56	0.58
Q9	0.53	Q25	0.69	Q41	0.52	Q57	0.66
Q10	0.72	Q26	0.78	Q42	0.56		
Q11	0.73	Q27	0.54	Q43	0.70		
Q12	0.57	Q28	0.54	Q44	0.83		
Q13	0.70	Q29	0.63	Q45	0.66		
Q14	0.51	Q30	0.69	Q46	0.84		
Q15	0.59	Q31	0.72	Q47	0.75		
Q16	0.74	Q32	0.70	Q48	0.74		

 https://doi.org/10.6018/educatio.432431

In Table 2, the larger the extraction commonalities are and closer they are to one, the better the extracted factors will explain the variables. The extraction commonalities value of most variables is greater than 0.40. Therefore, it was shown that there is no need to extract another factor and these factors (values above 40%) that will be extracted in the next steps explain more than 40% of the variable changes.

Table 3
Total variance explained

Component	Extraction Sums of Squared Loadings	Rotation Sums of Squared Loadings		
	Cumulative %	Total	% of Variance	Cumulative %
1	36.36	12.80	22.46	22.46
2	50.92	9.29	16.30	38.76
3	57.25	8.77	15.38	54.15
4	62.65	4.42	7.76	61.91
5	66.72	2.74	4.80	66.72

The results of Table 3 show that the first four factors have special values greater than one, and using this criterion we can say that if we consider the number of extraction factors as four factors, 66.72% of the variance of all variables will be explained. Then, considering the third output in Table 4, it can be observed that, the first part is related to the specific values of extraction factors without rotation and the second part is related to the specific values of extraction factors with rotation. The results of the rotation matrix are as follows:

Table 4
The rotation matrix

Summary of the statements	Factors				
	First	Second	Third	Fourth	Fifth
Understanding derivatives by focusing on details			0.84		
Understanding derivatives through basic rules			0.84		
Full understanding of derivatives according to the derivative's context and subject			0.75		
Judgment on answering derivative problems and their application			0.75		

Summary of the statements	Factors				
	First	Second	Third	Fourth	Fifth
Viewing the relationships between derivative and its multiple applications in one's field of study			0.71		
The importance of knowing why to use derivatives compared to knowing how to take derivatives			0.75		
Understanding derivatives through conceptual discovery of the concepts of limit and continuity			0.64		
Similarity of classmates' understanding and self-understanding of derivatives			0.57		
Overall insight into understanding the derivative definition of the concept of limit and other mathematical concepts			0.69		
Ability to explain the definition of derivative's mathematical understanding			0.84		
Ability to understand derivatives fully and definitively			0.84		
Gradual learning of derivative and its application			0.74		
Automatic understanding of the derivative's definition Understanding the derivative definition along with prior understanding of the relevant mathematical concepts			0.75 0.67		
The complexity of understanding the derivative's definition			0.74		
One's ability to make his own derivative questions	0.70				
Strengthening the understanding of derivatives by its explanation to the classmates	0.82				
Knowing the different ways to solve a practical problem of derivative	0.77				
Interacting with real and then abstract derivative subject matters	0.75				
Organizing the derivative concept through personal practices	0.73				
Practicing a lot of similar exercises to master derivatives	0.71				

 https://doi.org/10.6018/educatio.432431

Summary of the statements	Factors				
	First	Second	Third	Fourth	Fifth
Better understanding of the learner through self-discovery or self-discovery in learning of derivative understanding	0.86				
Insisting on memorizing derivative formulas	0.88				
Learner's confusion through different derivative teaching methods	0.87				
Instructor's explanation about the best way to understand derivatives	0.75				
Being aware of the derivative principles before solving its exercises	0.85				
Having knowledge about the history of derivative	0.57				
Emphasizing the accurate memorization of the derivative formulas	0.59				

Table 4
The rotation matrix

Summary of the statements	Factors				
	First	Second	Third	Fourth	Fifth
Facilitating the learning of derivative through book descriptions	0.71				
Preference of classmates' explanations over teacher's explanations about learning derivatives	0.79				
Improving personal skills in learning the derivative concepts	0.79				
simplifying the derivative concept	0.77				
Believing in multiple solutions to derivative applied problems	0.78				
Acquiring the knowledge of limit definition and slope of a tangent line as a prerequisite		0.71			
Having a positive attitude towards learning derivatives		0.57			
Lack of learners' efforts in learning derivatives		0.62			
Failure to succeed due to the lack of motivation towards understanding derivatives		0.65			

Summary of the statements	Factors				
	First	Second	Third	Fourth	Fifth
Uncertainty in working with derivatives in objective and real problems		0.61			
The need to have the visualization ability		0.64			
Concentrating and thinking to understand the definition of derivative		0.84			
Lack of innate ability in mathematics and its relation to the lack of derivative understanding		0.66			
Understanding derivatives apart from the verbal ability of the derivative concept		0.70			
Lack of understanding derivatives due to the existence of too many symbols and formulas		0.75			
The need to have a good memory to memorize the details of understanding derivatives		0.84			
Dependence of understanding derivatives and other concepts of calculus on people's intelligence		0.72			
Difficulties s in understanding mathematics at the Calculus level by the increase in age		0.84			
Hard work in understanding derivatives		0.73			
Different abilities to understand derivative's verbal problems and its applications					0.75
The similarity of understanding derivatives and its application with the understanding of other concepts of calculus					0.90
Understanding derivatives' efficiency and its application in understanding practical issues					0.89
Computational-conceptual problem				0.67	
Computational-conceptual problem				0.72	
Computational-conceptual problem				0.72	
Derivative's conceptual problem				0.73	
Derivative's computational problem				0.58	
Computational-conceptual problem				0.62	
Computational-conceptual problem				0.57	

https://doi.org/10.6018/educatio.432431

Table 4 shows the given rotation matrix of the components using the varimax rotation method. The given rotation matrix of the components includes the factor loads of each factor among the residual factors after rotation. Based on the above results, the factors (from first to the fifth) are named as follows:

- The first factor: Teaching - Learning about Derivative Understanding
- The second factor: conditions and barriers related to understanding derivatives
- The third factor: nature of understanding derivatives
- The fourth factor: relational and instrumental understanding
- The fifth factor: comparison of understanding derivative concept with other concepts

When the questionnaire is modified, the exploratory factor analysis (EFA) method should be used. The last step of this method is to the rotation matrix. In this matrix, the questions related to the components of the questionnaire, which we modified and specified after several steps, are fixed. In other words, in Table 4, or the rotated matrix, it is specified that this questionnaire has five components and what questions each component covers. Components with their questions can be a tool for measuring derivative understanding.

Discussion

Teaching for understanding is often considered as an important educational goal. Students should learn and understand mathematical ideas and principles, including techniques and skills in performing mathematical calculations. Derivative is one of the fundamental issues in calculus, which analyzes the way things change and the rate of their changes. Learners' difficulties and problems with many previous concepts, such as functions, and especially lack of dynamic insight into functional dependency, slope, velocity, ratio, rate of change, and limit are well documented. Studies that have focused specifically on learners' understanding of derivatives show a weakness in their conceptual understanding. The importance of conceptual understanding, along with mastery of procedures, has been emphasized as part of learners' mathematical skills.

Learners can learn separately without having the opportunity to make connections between concepts and basic relationships. This issue reduces the conceptual understanding of important concepts in mathematics. Learners can fill this gap with procedural understanding, and calculation can be considered as the main consequence of having little conceptual understanding. Moreover, learners have difficulty in conceptualizing and relating the rate of change to the concept of derivative. Another conceptual problem is related to the recognition of the difference between the average rate of change and the instantaneous rate of change in relating these concepts to the concept of derivative. Learners also have difficulty in conceptualizing the role of limit in providing an algebraic definition of derivative, understanding how the rate of change approaches the instantaneous rate of change, and understanding how the slope of intersecting lines approaches the slope of tangent lines. Relational understanding of derivative should include knowledge of the major fundamental ideas in the concept of derivative, that is, the rate of change, the slope of tangent line, and limit, and the relationships among them. Although learners can solve derivative problems correctly, they cannot explain the derivative by relating it to the rate of change, the slope of tangent line, and the limit. As one of the important reasons for this type of learning conflicts and academic success, many researchers emphasize the role of the memorizing process without understanding the major basic ideas. For most learners, derivative is formed of many unreasonable rules (i.e, instrumental understanding). From this point of view, it is necessary to consider the relational form of the derivative for the conceptual understanding of its concept.

In the discussion of understanding derivatives, especially its relational and instrumental types of understanding that was introduced and specified by Skemp years ago, no studies have been done and no research tools have been designed and provided to address these two issues. Therefore, lack of such a research instrument in the related studies is noticeable. Determining the level of relational and instrumental understanding, can in turn determine the different ways of learning calculus. Undoubtedly, learning the abstract concepts of calculus is challenging for the first-year university students, and the way they understand these concepts can pave the way for other ways of learning the concepts related to calculus. Regarding the concept of derivative, many students must use basic mathematical concepts in their early years at university, especially in the fields related to the domains of basic sciences, engineering, and humanities.

Even the students of management and economics, etc., need to learn the concept of derivative. Many students might learn the concept of derivative superficially, understand it to some extent instrumentally, and act superficially and formulaically in most concepts related to derivative, when they are in the early years of university. Instrumental understanding is evident in many students' responses. We cannot claim that understanding is a superficial, fleeting, and perhaps useless type of understanding. In fact, we can infer that instrumental understanding can be the basis for learning, but it must be conceptually changed. In order to achieve a relational and permanent understanding of the concept of derivative, students must go through a series of steps that interruption at any of these steps, will disrupt their understanding in this area. Therefore, relying on the above-mentioned justifications, it seems that a questionnaire consisted of several sections of textual and computational questions should be designed to approximately estimate the students' understanding of the derivative.

Conclusion

In this study, the researchers tried to modify and adapt Yoong's Mathematical Comprehension Questionnaire (1984) by relying on their goals and the previous studies in the fields of mathematical comprehension, and Skemp's instrumental and relational comprehension and the different perspectives, and direct their desired goals towards the understanding of derivative by relying on the components of understanding in this questionnaire. On the other hand, the researchers of this study tried to evaluate the learner's instrumental-relational understanding by studying the selected questions and issues adopted from several areas of derivative concept application (the emphasis is on understanding the derivative and not its use). Therefore, some questions from the two books of Hoffmann & Bradley (2010) and Thomas et al. (1988) were considered in the questionnaire. After examining the content validity, a few questionnaire items and some of derivative's computational-conceptual questions/items were removed. For this reason, the first-year university students were selected for this study, many of whom had to learn a basic understanding of derivative concept at a slightly higher level than high school level, and therefore, their professors were asked to introduce students who were at the desired academic level (their midterm scores in the mathematics course was desirable) to the researchers of this study. On the other hand, the computational-conceptual questions of derivative should have been evaluated based on the instrumental-relational understanding of the derivative concept based on the Skemp's point of view. Therefore, based on Skemp's theoretical foundations on relational and instrumental understanding, the researchers of this study designed relevant options in Likert scales in such a way that two options had some degrees of instrumental understanding and two others had some degrees of relational understanding and one option was considered for the case in which a student does not respond to the question. After removing a number of questions after checking the content validity, the questions were distributed among the sample, and finally, after modifying a number of item statements, we came up with five factors with specific items that are as follows: teaching-learning related to understanding derivative; conditions and barriers related to understanding derivatives; nature of understanding derivatives; relational and instrumental understanding; comparison of understanding derivative concept with other concepts. Therefore, based on this study, the researchers have considered suggestions for future research; an instrument can also be developed in the domain of derivative application based on the instrument designed and presented in this study. Based on the results of the instrumental and relational understanding components, the number of items can be increased, and similar questionnaire can be designed in other related sciences; we can also proceed towards the application stage of derivatives and design a test to evaluate the relational and instrumental understanding in a wider scope. It is also suggested to evaluate and analyze the instrumental and relational understanding of the selected students' professors. Moreover, we can identify barriers to learning derivatives by relying on factors that have influence on understanding the concept of derivative, and try to design and provide lesson plans which can help to minimize the students' challenges. Finally, it is better to consider this questionnaire for other topics of calculus and investigate the students' understanding of those subject matters.

 https://doi.org/10.6018/educatio.432431

References

Aksoy, Y. (2007). *The effect of computer algebra systems on the teaching of derivative concept*. Ph.D. Thesis. Gazi University, Ankara.

Ayebo, A., Ukkelberg, S., & Assuah, C. (2017). Success in introductory calculus: The role of high school and pre-calculus preparation, *International Journal of Research in Education and Science (IJRES)*. 3(1), 11-19.

Bahar, E. E., Rahman, A., & Minggi, I. (2012). Analisis Pemahaman Mahasiswa Terhadap Konsep Limit Fungsi di Satu Titik (Studi Kasus pada Mahasiswa Jurusan Matematika FMIPA UNM). *Sainsmat*, 1(2), 181–190.

Balcı, M. (2012). *Genel matematik-1*. İzmir: Sürat Üniversite Yayınları.

Bingölbali, E. (2010). Türev kavramına ilişkin öğrenme zorlukları ve kavramsal anlama için öneriler. M.F. Özmantar, E. Bingölbali ve H. Akkoç (Ed.), *Matematiksel kavram yanılgıları ve çözüm önerileri* (Gözden geçirilmiş baskı) içinde (223-255). Ankara: Pegem Akademi yayınları.

Berresford, G. & Rockett, A. (2015). *Brief applied calculus*. Nelson Education.

Bressoud, D.M., Mesa, V., & Rasmussen, C. (2015). *Insights and recommendations from the MAA National Study of College Calculus*. New York: MAA Press.

Bouguerra, R. (2019). Students' difficulties to learn derivatives in the Tunisian context. In Eleventh Congress of the European Society for Research in Mathematics Education (No. 08).

Denbel, D.G. (2015). Some Conceptual Difficulties of Students on Derivation. *Journal of Educational and Management Studies*, 5(4), 211-214.

Desfitri, R. (2016). In-Service Teachers' Understanding on the Concept of Limits and Derivatives and the Way They Deliver the Concepts to Their High School Students, *Journal of Physics: Conference Series* 693, 012016, 1-9.

Fuentealba, C., Badillo, E., Sánchez-Matamoros, G., Cárcamo, A. (2019). The Understanding of the Derivative Concept in Higher Education. *EURASIA Journal of Mathematics, Science and Technology Education*, 15(2), 2-15.

Fuentealba, C., Badillo, E., Sánchez-Matamoros, G. (2019). Identificación y caracterización de los subniveles de desarrollo del esquema de la derivada. *Enseñanza de las Ciencias*, 37(2), 63-84.

Gür, H., & Barak, B. (2007). Ortaöğretim 11. sınıf öğrencilerinin türev konusundaki hata örnekleri. *Educational Sciences: Theory & Practice, 7*(1), 453-480.

Haapasalo, L. & Zimmermann, B. (2015). *Investigating mathematical beliefs by using a framework from the history of mathematics*. In: C. Bernack-Schler, R. Erens, T. Leuders & A. Eichler (Eds.), *Views and beliefs in mathematics education* (pp. 197–211). Wiesbaden, Germany: Springer Spektrum.

Hähkiöniemi, M. (2005). Is there a lımıt in the derıvatıve? – exploring students' understanding of the limit of the difference quotient. In M. Bosch (Eds), *Proceedings of the fourth congress of the European society for research in mathematics education* (CERME 4), Sant Feliu de Guíxols, Spain, 17 – 21 February 2005, 1758-1767.

Hoffmann, L.D. & Bradley, G.L. (2010). *Calculus for Business, Economics, and the Social and Life Sciences*, Claremont McKenna College.

Jaworski, B. (2015). Mathematics meaning-making and its relation to design of teaching. *PNA*, 9 (4), 261–272.

Karadeniz, A. A. (2003). *Yüksek matematik cilt-1 diferansiyel ve integral hesap*. Çağlayan Basımevi. İstanbul. 13. Baskı.

Kilpatrick, J. Swafford, J. & Findell, B., eds. (2001). *Adding it up: Helping children learn mathematics*, Washington DC: National Research Council.

Laridon, P., Jawurek, A., Kitto, A., Pike, M., Myburgh, M., Rhodes-Houghton, R., Sasman, M., Scheiber, J., Sigabi, M., & Van Rooyen, R. (2007). *Classroom Mathematics: Grade 12 learners' book*. Heinemann Publishers (Pty) Ltd, Johannesburg: South Africa.

Lee, J. (2018). An Analysis of University Students' Representational Fluency About the Derivative Concept. *The journal of educational research in mathematics*, 28(4), 573-598.

Maharaj, A., & Ntuli, M. (2018). Students' Ability to Correctly Apply Differentiation Rules to Structurally Different Functions. *S Afr J Sci*, 114(11/12), 1-7.

NCTM (National Council of Teachers of Mathematics). (2014). *Principles to actions: Ensuring mathematical success for all*. Reston, VA: NCTM.

Özkan, E.M. & Ünal, H. (2009). Misconception in Calculus-I: Engineering students' misconceptions in the process of finding domain of functions. *Procedia - Social and Behavioral Sciences*, 1(1), 1792-1796.

Pino-Fan, L. R., Godino, J. D., & Font, V. (2018). Assessing key epistemic features of didactic-mathematical knowledge of prospective teachers: the case of the derivative. *Journal of Mathematics Teacher Education*, 21(1), 63-94.

Roundy, D. Dray, T. Manogue, C. A. Wagner, J. & Weber, E. (2015). *An extended theoretical framework for the concept of the derivative*. In T. Fukawa-Connelly, N. E. Infante, K. Keene, & M. Zandieh (Eds.).

Rohde, U. L. Jain, G. C. Poddar, A. K. & Ghosh, A. K. (2012). *Introduction to differential calculus: systematic studies with engineering applications for beginners*. Wiley.

Sahin, A., Cavlazoglu, B., & Zeytuncu, Y.E. (2015). Flipping a college calculus course: A Case study. *Journal of Educational Technology & Society*, 18(3), 142.

Sahin, Z., Yenmez, A.A., & Erbas, A. (2015). Relational Understanding of the Derivative Concept through Mathematical Modeling: A Case Study, *Eurasia Journal of Mathematics, Science & Technology Education,* 11(1), 177-188.

Sánchez-Matamoros-Garcia, G., García Blanco, M.M. y Llinares Ciscar, S. (2008). La comprensión de la derivada como objeto de investigación en didáctica de la matemática. Revista Latinoamericana de Investigación en Matemática Educativa, 11 (2), 267-296.

Skemp, R. R. (1976). Relational understanding and instrumental understanding, *Mathematics Teaching, 77*, 20–26.

Skemp, R.R. (1978). Relational understanding and Instrumental understanding. *Arithmetic Teacher*, 26(3), 9-15.

Smith, S. R. (2015). Incomplete understanding of concepts: The case of the derivative. *Mind*, 124(496), 1163-1199.

Star, J. R. & Stylianides, G. J. (2013). *Procedural and Conceptual Knowledge: Exploring the Gap Between Knowledge Type and Knowledge Quality*. Canadian.

 https://doi.org/10.6018/educatio.432431

Thomas, G.B. D. Weir, M., & Hass, J. (1988). *Thomas' Calculus*, 11th Edition, Massachusetts.

Tierney, W. G., & Garcia, L. D. (2008). Preparing Underprepared Students for College: Remedial Education and Early Assessment Programs. *Journal of at-Risk Issues*, 14(2), 1-7.

Van Engen, H. (1953). The formation of concepts. In H.F. Fehe (Ed.), *The learning of mathematics: Its theory and practice*, 21st yearbook of the National Council of Teachers of Mathematics. Washington, DC: The Council, 69-98.

Watson, A, & Sullivan, P. (2008). Teachers learning about tasks and lessons. In D. Tirosh, & T. Wood (Eds), *Tools and resources in mathematics teacher education* (pp. 109-135). Rotterdam: Sense Publishers.

Widada, W., Sunardi, H., Herawaty, D., Boby, E., & Syefriani, D. (2018). Abstract Level Characteristics in SOLO Taxonomy during Ethnomathematics Learning, *Int. J. Sci. Res.*, 7(8), 352–355.

Woods, R.G. & Barrow, R.C. (1975). *An introduction to philosophy of education*. London: Methuen.

Yoong, W.KH. (1984). Aspects of mathematical understanding, *Singapore Journal of Education*, 8(2), 45-55.

Appendix

Please answer the following questions and state your reasons for answering each question. Note that even if you use the formula, state the reason for your use and the answer.

51) If the function *p(t)* represents the population of a community at time *t* of the following year, in this case, what does *p'*(5)=1000 and *p'*(10)= -2000 mean?

52) Experiments show that when a rare type of insect jumps, the jump height (in meters) after *t* seconds is a function of the following:

$$H(t)=4/4t - 4/9\ t^2$$

A) First find *H'(t)*. What is the rate of change of *H(t)* after one second?
B) When is *H'(t)* = 0? What matters during this time?

53) Determine and explain the derivative of *f(x)=sinx* and *g(x)=cosx* functions by defining the derivative limit.

54) Suppose the absolute value function *f(x)* is:

$$f(x) = \begin{cases} x x \geq 0 \\ -x x < 0 \end{cases}$$

Show that:

$$f'(x) = \begin{cases} 1 x \geq 0 \\ -1 x < 0 \end{cases}$$

And state why this function is not derived from *x* = *0*?

55) Find the derivative of $f(x) = \frac{(x+1)h(x)}{(2x+1)h(2x+1)}$ at the $x = -1$.

56) A particle is moving in a horizontal direction with the following positive function:

$$s(t) = 2t^3 - 14t^2 + 22t - 5t \geq 0$$

Find the velocity and acceleration and describe the motion of this particle in time intervals *t* = 0 and *t* = 11/3.

57) What is the behavior of a curve $f(x) = \sqrt[3]{x^3 - 2x^2 + x}$ near a point of *x* = 1?

 https://doi.org/10.6018/educatio.432431