

Elegance and Parsimony in First-Order Necessitism

Elegancia y parsimonia en el necessitismo de primer orden

VIOLETA CONDE¹

In his book *Modal Logic as Metaphysics*, Timothy Williamson defends first-order necessitism using simplicity as a powerful argument. However, simplicity is decomposed into two different, even antagonistic, sides: elegance and parsimony. On the one hand, elegance is the property of theories possessing few and simple principles that allow them to deploy all their theoretical power; on the other hand, parsimony is the property of theories having the fair and necessary number of ontological entities that allow such theories give an account of themselves. Since necessitism endorses Barcan Formulae for the sake of elegance, it is committed to a vast number of contingently non-concrete objects, so one may think that it is not qualitatively parsimonious. I argue that necessitism could be viewed as an *additive* case in the sense that Alan Baker characterizes the adjective, so quantitative parsimony should be considered when it comes to necessitism instead of qualitative parsimony.

Keywords: necessitism, elegance, parsimony, Barcan formulae, contingently non-concrete objects

En su libro *Modal Logic ad Metaphysics*, Timothy Williamson defiende la doctrina necessitista de primer orden aludiendo a la simplicidad como un poderoso argumento. Sin embargo, la simplicidad puede ser caracterizada ateniéndonos a dos facetas diferentes e, incluso, opuestas: elegancia y parsimonia. Por un lado, la elegancia sería la cualidad de las teorías de poseer unos pocos principios simples a partir de los cuales desplegar todo su poder teórico, mientras que, por otro, la parsimonia sería la propiedad de las teorías de contar con el número justo y necesario de entidades para dar cuenta de sí misma. El necessitismo acepta las fórmulas Barcan en pos de la elegancia, pero la aceptación de las Fórmulas Barcan conlleva el compromiso con un gran número de objetos contingentemente no concretos, por lo que se podría pensar que no es cualitativamente parsimonioso. Defenderé que el necessitismo puede considerarse como un caso *aditivo* en el sentido en el que Alan Baker caracteriza tal adjetivo y que, por tanto, es preciso tener en cuenta la parsimonia cuantitativa en lugar de la cualitativa en lo que a esta tesis respecta.

Palabras clave: necessitismo, elegancia, parsimonia, fórmulas Barcan, objetos contingentemente no concretos

0. Introduction

Simplicity has often been considered a strong *desideratum* when establishing a philosophical or scientific system. Allusions to simplicity as a desirable property are found early in the literature, for Aristotle, in his *Posterior Analytics*, asserts that "we may assume the

Recibido: 01/05/2023. Aceptado: 17/10/2023.

¹ La autora es Investigadora Predoctoral en Formación FPU en la Universidad de Santiago de Compostela. Correo electrónico: violeta.conde.borrego@usc.es. Líneas de investigación: metafísica y epistemología de la modalidad, lógica modal, filosofía de la lógica. Publicaciones destacadas: Conde, V. (2023) [Forthcoming]. "Necessitism and Unrestricted Quantification", *Teorema*, Vol. XLII/2. Esta investigación ha sido posible gracias al apoyo del Ministerio de Universidades de España a través del programa nacional FPU (referencia de la ayuda: FPU19/00199) y se inscribe en el Proyecto de Investigación "Concepciones deflacionarias en ontología y metaontología" (referencia: PID2020-115482GB-I00).

superiority, *ceteris paribus*, of the demonstration which derives from fewer postulates or hypotheses" (Aristotle, 1941: 150). Moreover, the principle of simplicity or economy has been called "Ockham's razor" and can be found in many other authors—such as Aquinas (1945), Newton (1964), Kant (1950), or Duhem (1906), to name a few—who have seen simplicity even as a feature of truth: the simpler a theory is, the more likely it is to be true. In his book *Modal Logic as Metaphysics* (2013), Timothy Williamson stands for second-order S_5 axiomatized with Barcan Formulae as the best logical system for explaining metaphysical modality. In that context, he refers to simplicity as an argument favoring first-order necessitism as a metaphysical theory of modality. But first, what is necessitism? As Williamson himself puts it:

[...] necessitism says that necessarily everything is necessarily something; still more long-windedly: it is necessary that everything is such that it is necessary that something is identical with it. In a slogan: ontology is necessary (Williamson, 2013: 2).

Another form of articulating what is necessitism is using the following formula, also called the “necessary necessity of being”:

$$(NNE) \Box \forall x \Box \exists y (x = y)$$

Moreover, as Williamson shows (Williamson 2013: 38), one could derive the necessary necessity of being (NNE), the principle at the core of necessitism, using Barcan Formulae. Barcan Formulae are adopted within the framework of first-order necessitism for simplicity, but they make necessitism to be committed with a vast number of *mere possibilia* or contingently non-concrete objects. I should first clarify what I mean by the expression “Barcan Formulae”. Usually, Barcan Formula (BF) is the name for the axiomatic schema $\Box \exists v A \rightarrow \exists v \Box A$ (or, equivalently, the following schema $\forall v \Box A \rightarrow \Box \forall v A$), while the schema $\exists v \Box A \rightarrow \Box \exists v A$ (or equivalently, $\Box \forall v A \rightarrow \forall v \Box A$) is known as the Converse Barcan Formula (CBF). These formulae were presented for the very first time by Ruth Barcan (Marcus) (1946) in a paper entitled *A Functional Calculus of First Order Based on Strict Implication* and had a major impact in the connections between logic and metaphysics. As we can see, accepting $\Box \exists v A \rightarrow \exists v \Box A$ commits us with *mere possibilia* or contingently non-concrete objects, for if it possible that there exists a v such as it is A , then there exists a v such it is a possible A . For instance, if it is possible that there exists the king of France, then there exists a possible king of France.

Contingently non-concrete objects exist just logically; they do not occupy any space or time. They could have been concrete, but contingently, they are not. In Williamsonian terminology, they L-exist:

In one sense it is obviously contingent what exists. This table might not have existed. But ‘exist’ has more than one sense. For in one sense events do not exist, they occur. Three-dimensional physical objects exist when they are somewhere. Call that the substantial sense of ‘exist’ (‘S-exist’), since we might conjecture that only substances exist in that sense (in some sense of ‘substance’). In another sense events do exist, simply because there are events; to exist is to be something. Call that the logical sense of exist (‘L-exist’), since it is definable given identity and the unrestricted quantifier. Trivially, everything L-exists; not everything S-exist because events do not (Williamson, 2000: 194).

One may think that a theory that is committed with such a huge number of contingently non-concrete objects is not parsimonious. However, what is meant by non-parsimonious in this context is far from clear.

In the first section of this paper, I will introduce the concepts of elegance and parsimony concerning simplicity; then (section 2), I will explore how these concepts apply to the necessitist framework and how, apparently, Williamson gives more weight to elegance than parsimony; finally (section 3), I will stand for necessitism to be an *additive* theory in the sense Baker (2003) characterizes the adjective; further I will explain the difference between qualitative and quantitative parsimony and I will argue that parsimony has the same weight as elegance to establish first-order necessitism when considered quantitatively instead of qualitatively.

1. Elegance and parsimony

The search of simplicity can be justified *a priori* by theological or philosophical reasons. It can be also justified by its intrinsic value, as Sober claims:

Just as the question ‘why be rational?’ may have no non-circular answer, the same may be true of the question ‘why should simplicity be considered in evaluating the plausibility of hypotheses?’ (Sober, 2001: 19).

Be as it may, simplicity has two faces, which are sometimes opposed. On the one hand, elegance is the property of theories with few simple principles from which the theory can deploy its whole potential; on the other hand, parsimony is the property of a theory with the

fair and necessary number of ontological entities. Russell (1951) and Quine (1966) claim that when an existential claim is removed from a theory—when the theory is more parsimonious—, the theory has more probability of being true since a conjunction always is going to have less probability of being true than its separated conjuncts.

Although elegance and parsimony could conflate, it is essential to acknowledge these two sides of simplicity, for many times, the realization of one of them is inversely proportional to the consecution of the other: to postulate extra entities in a theory could make the theory simpler, whereas to reduce the ontology of theory could be possible only at the expense of making the syntax of the theory more complex (see Baker, 2022). Likewise, we should not mislead dispensing with entities for the sake of parsimony with the reduction of some entities to others. When a reduction is reached by establishing an identity relation, we are not dispensing with entities since the number of fundamental entities remains unaffected; for instance, reducing numbers to sets and dispensing with phlogiston are two very different cases: in the first case, the number of entities in our theory remains the same, for we have just reduced some entities—numbers—to another class of entities—sets—, whereas in the second case we have made disappear a theoretical entity from our theory, since its explanatory role is not useful anymore.

2. Williamson on elegance and parsimony

Contingentists² argue that the reading of Barcan Formulae is not intuitive, and they provide examples to support that thesis. I will present two of them, one for BF and another one for CBF, that could be found in Williamson’s work (Williamson, 2013).

Let’s begin considering the counterexample to BF. Consider the formula that results from replacing A in *full* BF³ by the open formula $x = y$:

(1) $\Box(\Diamond\exists x x = y \rightarrow \exists x \Diamond x = y)$, where y is an arbitrary material object.

² Contingentism is the view that denies necessitism.

³ By *full* BF I mean the schema whose instances are all modal closures of instances of BF, i.e. $\Box(\Diamond\exists v A \rightarrow \exists v \Diamond)$.

Since the formula $\exists x x = y$ is true at least for one value of y , one can derive $\Box \Diamond \exists x x = y$ (considering that the actual is necessary possible), and, from there, we can arrive—by (1) and standard modal reasoning—to the formula $\Box \exists x \Diamond x = y$. However, the former formula is false for contingentists, since they think that it is not necessary that there exists something that could be y . A necessitist, for the contrary, can accept something that could be y : a contingently non-concrete object.

Consider now the formula that results from replacing A by the formula $\neg \exists y x = y$ in CBF:

$$(2) \exists x \Diamond \neg \exists y x = y \rightarrow \Diamond \exists x \neg \exists y x = y$$

For a contingentist, any ordinary object would satisfy the antecedent of (2)—there is something that could have been nothing— but the consequent would be false, since $\exists x \neg \exists y x = y$ (something is nothing) is inconsistent in first-order logic with identity. But if we apply *modus tollens* to (2) we obtain (NNE), which shows, according to Williamson (2013: 38) that Barcan Marcus’ system is necessitist in spirit.

The main task for the contingentist, then, is to identify a fallacy in the proof of the relevant instance of *full* CBF.⁴ That proof implies the following formula:

$$(3) \Box (\neg \exists y x = y \rightarrow \exists x \neg \exists y x = y),$$

which can be understood as the implicit assertion of the following universal sentence:

$$(4) \forall x \Box (\neg \exists y x = y \rightarrow \exists x \neg \exists y x = y).$$

For a contingentist, (4)—and, therefore, (3)—cannot be accepted; even if the king of France had been nothing, this would not imply that there was something that is nothing.

Taking into account the above, we have that the contingentist does not accept (3), this being the necessitated version of (5):

⁴ *Full* CBF is the schema whose instances are all modal closures of instances of CBF, i.e. $\Box \exists v \Diamond A \rightarrow \Diamond \exists v$

$$(5) \neg\exists y x = y \rightarrow \exists x \neg\exists y x = y.$$

Therefore, if she wants to keep the principle (NNE), she must sacrifice the validity of (5); that is, our contingentist has to choose between rejecting (5) as a theorem or rejecting (NNE)—which leads from (5) to (3). Since (5) is a standard non-modal logic theorem— that is, an instance of the existential generalization principle $A \rightarrow \exists vA$ —the first alternative would imply the adoption of a free logic (even for the non-modal fragment) that imposes restrictions on quantification. If she chooses, instead, to restrict the principle (NNE)—that is, to accept (5) as a theorem but not (3)—, then, she would be renouncing the validity of a large number of open formulas, alluding that such formulas “lack fixed interpretations and so serve a merely instrumental role in the proof theory and semantics” (Williamson, 2013: 40).

Adopting free logic, even for the non-modal fragment of our language, seems not to be very elegant for Williamson. Instead, he proposed adopting Barcan Formulae as axioms since that simplifies the logic for his theory about modality. However, adopting Barcan Formulae commits him to contingently non-concrete objects, a new ontological category of objects whose existence is controversial and counterintuitive. In that sense, necessitism would not be parsimonious, at least qualitatively (for it postulates a new kind of entities). In the next section, I will argue that, even if necessitism is not parsimonious in the qualitative sense, it is so in the quantitative sense.

3. Quantitative parsimony

We can differentiate between qualitative and quantitative parsimony. Qualitative parsimony refers to the kind of things a theory postulates; namely, the less variety of kinds, the more qualitatively parsimonious the theory is. On the other hand, quantitative parsimony concerns the number of things a theory is committed to. Many philosophers regard quantitative parsimony as non-significative when constructing their metaphysical theories. However, there are cases in science for which quantitative parsimony has shown to be important. Following Daniel Nolan (1997) in his paper *Quantitative Parsimony*, let’s consider what he calls the “neutrino case”. The neutrino was posed as an entity to explain some phenomena that occurred during the so-called Beta decay. However, as Baker (2003) points out those phenomena could be explained by different neutrino hypotheses, namely:

(H₁) 1 neutrino with a spin of $\frac{1}{2}$ is emitted in each case of Beta decay.

(H₂) 2 neutrinos, each with a spin of $\frac{1}{4}$ are emitted in each case of Beta decay.

(H₃) 3 neutrinos, each with a spin of $\frac{1}{6}$ are emitted in each case of Beta decay.

... and more generally, for any positive integer n ...

(H _{n}) n neutrinos, each with a spin of $\frac{1}{2n}$ are emitted in each case of Beta decay.

The neutrino case is what Baker (2003) calls an *additive* case:

My analysis in this paper is restricted to a class of cases I shall refer to as *additive*. Such cases involve the postulation of a collection of individual objects, qualitatively identical in the relevant respects, which collectively explain some particular observed phenomenon. The explanation is ‘additive’ in the sense that the overall phenomenon is explained by totaling the individual positive contributions of each object. I shall argue that in additive cases such as the neutrino case, it is rational to prefer quantitatively parsimonious hypotheses, not because quantitative parsimony is a primitive theoretical virtue, but because quantitative parsimony brings with it other independently recognized virtues. In particular, quantitative parsimony tends to increase the explanatory power of hypotheses compared to their less quantitatively parsimonious rivals (Baker, 2003: 248)

According to Baker (2003), quantitative parsimony should be considered instead of qualitative parsimony in cases which are *additive*, such as the neutrino one. The history of science has shown us that considering this kind of parsimony in additive cases has been more fruitful than considering less quantitatively parsimonious accounts. Nolan considers this way of thinking a meta-induction procedure: “the fact that employing quantitative parsimony has led to more accurate theories in the past is evidence that it will in the future also” (Nolan 1997: 332). In the neutrino case, considering (H₁) has the same effect than considering, for instance, (H₂) since the Beta decay phenomenon is totally explained by the former hypothesis. However, we can say that considering (H₁) make the theory simpler and more manageable.

Necessitism is qualitatively non parsimonious since it postulates a new kind of objects as a result of the addition of the Barcan Formulae as axioms for S₅. Williamson seems not to be

very concerned about parsimony as a virtue in the case of necessitism.⁵ However, I will argue that if some contingentist charges necessitism on the base of not to be parsimonious, a necessitist can answer saying that it is so in the quantitative sense, for it does not multiply contingently non-concrete objects without necessity. It is true that the number of contingently non-concrete objects is huge, but they are all there in the logical space from the beginning, so no additional objects are postulated: the number of objects postulated is the fair and necessary one; the fact that necessitism postulates all of them could be considered just as a limit case when referring to quantitative parsimony.

I start clarifying when a metaphysical explanation is additive. By analogy to scientific explanation, we can say that a metaphysical explanation is additive when the objects involved are qualitatively identical in the relevant respect and collectively serve to account for a metaphysical fact. Are contingently non-concrete objects qualitatively identical? Yes, they are since they are all contingently non-concrete and the other properties they have are non-qualitative ones.⁶ Do they explain collectively a metaphysical fact? They explain the overall phenomenon of metaphysical modality in a sense in which *existence* is understood logically and not substantively. So, as I argue below, quantitative parsimony is preferable than qualitative parsimony in the case of necessitism.

The final question will be: is it necessitism, as Williamson poses it, quantitatively parsimonious? I will say yes, because it does not duplicate contingently non-concrete objects without reason, since, as I have said, they are all there in the logical space from the beginning. Contingently non-concrete objects, as Linsky and Zalta (1996) point out, play a fundamental role in some modal claims; they guarantee the truth of the consequent of the Barcan Formula, and, as Williamson and I argue, using Barcan Formulae makes necessitism a more elegant theory about metaphysical modality than contingentism.

The explanatory power that necessitism has shown concerning the metaphysics of modality seems enough to accept contingently non-concrete objects as a new kind of entity. Moreover, even if necessitism could be charged of not being qualitatively parsimonious, is quantitative

⁵ See Williamson, 2013.

⁶ I argue elsewhere that contingently non-concrete objects are tracked by their non-qualitative properties, in particular, by their haecceities.

parsimony what plays a more interesting role in the case of necessitism, so we can maintain that necessitism is elegant and parsimonious, at least in the quantitative sense.

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