

Mathematics as Activity

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ABSTRACT

What is Mathematics in itself? The Action Theory can contribute to answering this question following two directions: i) looking at *mathematical activity*; ii) putting the touchstone in this Science as *activity*, a type of action among others. In the first case, Mathematics receives a «direct study»: the research is carried out in order to make explicit the features of an autonomous activity; in the second case, the investigation is made in terms of an «indirect study»: it requires the connection and comparison with other actions.

Firstly, the task is to clarify the distinction between *mathematical activity* and Mathematics as *activity*, using the duality «act» (the inner element) and «action» (the outer component) within the field of «activity». Secondly, the character of mathematical actions is studied: a) as meaningful activity, Mathematics needs a purpose; b) the status of mathematical actions is specific, different from the activities developed in other Sciences; and c) the «internal factors» have more weight than the «external factors» in Mathematics. Thirdly, the description of mathematical actions contributes to completing the features of this specific activity.

A common characteristic of the great foundationalist programmes in Mathematics is to emphasize the non-empirical character of this Science. They give it a specific *status*, even when Mathematics is understood as a consequence of Logic. But these programmes reveal strong differences when trying to clarify *what* Mathematics is. The divergences are particularly relevant in crucial points like the problem of what its subject-matter is and how to understand the proofs. In this respect, Logicism, Intuitionism and Forma-

lism have followed different paths from the very beginning: the starting points are quite different.

In spite of the differences among them, it is a custom to put all these programmes together under the label of «followers of Mathematics as Formal Science». *De facto*, they agree in some ways: Mathematics requires a specific Semantics – a formal one –, the mathematical knowledge is a priori¹ and its progress follows a different Methodology from the other Sciences; hence Mathematics has an independent development from Empirical Sciences. Later, after the general criticism of the foundationalist programmes, the emphasis is removed from this original place. We can appreciate the change in authors like L. Wittgenstein, who has a strong attraction for empiricism in Mathematics, and I. Lakatos, who proposes a «quasi-empiricism» with a basis in the History of Science. Nevertheless, neither the former nor the latter adopts the radical view: Mathematics as Empirical Science. They take another new step: in the strict sense, there is no necessity to look for a «foundation».

With the new positions the main question remains: what is Mathematics in itself? The Action Theory can help us to look at *mathematical activity* or to adopt another posture: to put the touchstone in this Science as *activity*, a type of action among others. In the first case, Mathematics receives a «direct study»: the research is carried out in order to make explicit the features of an autonomous activity, like the intuitionist conception of Mathematics as a free creation of the mind. Meanwhile, in the second case, the investigation is made in terms of an «indirect study»: it requires the connection and comparison with other actions (like in the Wittgensteinian remarks), insofar as the activity of this Science is, among other facets, more one of human beings. From the point of view of the content of the relation between Mathematics and «activity», the convergence of pro-foundationalist and anti-foundationalist positions is possible. In fact, L. E. J. Brouwer has some similarities with Wittgenstein.

However, the differences are important, especially because Brouwer constructs Mathematics as «an essentially languageless activity of the mind»². In his edifice of mathematical thought, language is only a «technique for memorizing mathematical constructions, and for communicating them to others, so that mathematical language by

1 «Virtually every philosopher who has discussed Mathematics has claimed that our knowledge of mathematical truths is different in kind from our knowledge of the propositions of the Natural Sciences. This almost unanimous judgment reflects two obvious features of Mathematics. For the ordinary person, as for the philosopher, Mathematics is a shining example of human knowledge, a subject which can be used as a standard against which claims to knowledge in other areas can be measured. However, this knowledge does not seem to grow in the same way as other bodies of human knowledge. Mathematicians do not seem to perform experiments or to wait the results of observations. Thus there arises the conviction that mathematical knowledge must be obtained from a source different from perceptual experience», KITCHER, PH.: *The Nature of Mathematical Knowledge*, Oxford University Press, Oxford, 1984, p. 3.

2 BROUWER, L. E. J.: *Cambridge Lectures on Intuitionism*, edited by D. van Dalen, Cambridge University Press, Cambridge, 1981, p. 4. BROUWER, L. E. J.: «Historical background, Principles and Methods of Intuitionism», *South African Journal of Science*, v. 49, (1952), p. 141.

itself can never create new mathematical systems»³. Wittgenstein is against this alinguistic mental construction insofar as he defends a radical publicity of language and the relevance of the actions of a human being performing Mathematics: he offers an explanation of Mathematics as a dominion – a mastery – of a human capacity that depends on language and is an element of forms of life. He cannot accept that the inner experience reveals how Mathematics can be rebuilt in a suitably modified form. Hence, the starting points are quite different⁴.

1. MATHEMATICAL ACTIVITY AND MATHEMATICS AS ACTIVITY

By using G. H. von Wright's distinction between the inner aspect of activity and the outer aspect of activity⁵, which I consider preferable to express as the duality «act» (the inner element) and «action» (the outer component), both within the field of «activity», we can find clear differences: Brouwer defends Mathematics as *act*, whereas for Wittgenstein Mathematics is a sort of *action* under the mediation of, or intermediated by, language. The intuitionist maintains the mental nature of the Mathematical activity as the important thing, and linguistic expression is only «efficient, but never infallible or exact»⁶: it is a mere codification of mental content. In contrast, the Wittgensteinian position includes a radical publicity of Mathematics and its language; moreover, the language makes it possible to grasp Mathematics. Wittgenstein highlights Mathematics as a type of language-games that relates to a *way* of living. Mathematical propositions depend on actions of human beings (like calculating, counting, measuring, ...): there is a connection between a mathematical concept and a determined *activity* in human life⁷.

Wittgenstein is a long way from Brouwer's idea of «the inevitable inadequacy of

3 BROUWER, L. E. J.: *Cambridge Lectures on Intuitionism*, p. 5. BROUWER, L. E. J., «Historical background, Principles and Methods of Intuitionism», p. 141.

4 Cf. GONZALEZ, W. J.: «Matemática intuicionista y lenguaje», *Anuario filosófico*, v. 18/2, (1985), pp. 179-189. GONZALEZ, W. J.: «Semántica Anti-Realista: Intuicionismo Matemático y Concepto de Verdad», *Theoria*, v. 12-13, (1990), pp. 149-170.

5 Cf. WRIGHT, G. H. VON: *Explanation and Understanding*, Cornell University Press, Ithaca, 1971, p. 87.

6 Cf. *Cambridge Lectures on Intuitionism*, p. 5; «Historical background, Principles and Methods of Intuitionism», p. 141. The negative conception of language appears in his first book: «Language, the slave of the illusion and reality, cannot be an instrument of truth ...», BROUWER, L. E. J.: *Leven. Kunst en Mystiek*, J. Waltman, Delft, 1905, p. 40; quoted and translated in STIGT, W. P. VAN, «L. E. J. Brouwer, The Signific Interlude», in TROELSTRA, A. S. and DALEN, D. VAN (eds): *The L. E. J. Brouwer Centenary Symposium*, North-Holland, Amsterdam, 1982, p. 505.

7 Cf. WITTGENSTEIN, L., *Bemerkungen über die Grundlagen der Mathematik*, edited by G. H. von Wright, R. Rhees and G. E. M. Anscombe, Suhrkamp, Frankfurt, 1974, VII, 24. Translated by G. E. M. Anscombe: *Remarks on the Foundations of Mathematics*, 3rd ed., Blackwell, Oxford, 1978, p. 390. Cf. *Bemerkungen...*, VII, 42/ *Remarks...*, p. 409.

language as a mode of description and communication»⁸. From the point of view of Action Theory, he is laying aside the programme for the foundation of Mathematics based on mental acts and rejecting the primacy of something previous to linguistic activity connected with human actions. So, Mathematics does not need to discover specific mental acts (Brouwer's first act and second act of Intuitionism). Generally speaking, «the *mathematical* problems of what is called foundations are no more the foundations of Mathematics for us than the painted rock is the support of a painted tower»⁹. The only «foundation» of Mathematics is our mathematical practice and this does not need to have its roots in another place.

Brouwer finds inspiration in the intuition of temporal succession. He considers «the falling apart of moments of life into qualitatively different parts, to be reunited only while remaining separated by time, as the fundamental phenomenon of the human intellect, passing by abstracting from its emotional content into the fundamental phenomenon of mathematical thinking, the intuition of the bare two-oneness. This intuition of two-oneness, the basal intuition of Mathematics, creates not only the numbers one and two, but also all finite ordinal numbers»¹⁰. Wittgenstein's emphasis lies in his interest in the rules of language use developing Mathematics and the empirical applications of mathematical propositions. His posture is not the result of possessing a relevant «intuition», because he adopts a more general position: the mathematical practice of human beings.

Mathematical activity in Brouwer is different from Wittgenstein's conception of Mathematics as *activity*. Brouwer's focus has a mentalistic starting point, where the mathematical construction is introspective and the concept of «proof» has a strong character, because we must have a proof in order to accept a proposition. His constructivism does not depend merely on the system of operations in which the proposition is used. Brouwer's conception of mathematical activity leads to a certain ontological description: there are *mathematical objects* created by sequences of mental acts. Wittgenstein maintains a quite different position, because his concept of «proof» does not have a strong character, and he rejects a mathematical Ontology proceeding from sequences of mental acts: Mathematics is a human activity – a sort of action –, but it is not mentalistic¹¹, and there is no necessity to believe in mathematical objects in order

8 *Cambridge Lectures on Intuitionism*, p. 5; «Historical background, Principles and Methods of Intuitionism», p. 141. In the construction of denumerable sets, «neither the ordinary language nor any symbolic language can have any other role than that of serving as a nonmathematical auxiliary, to assist the mathematical memory or to enable different individuals to build up the same set», BROUWER, L. E. J., «Intuitionism and Formalism», *Bulletin of the American Mathematical Society*, v. 20, (1913); reprinted in BENACERRAF, P. and PUTNAM, H. (eds): *Philosophy of Mathematics*, B. Blackwell, Oxford, 1964, p. 81.

9 *Bemerkungen...*, VII, 16/ *Remarks...*, p. 378.

10 BROUWER, L. E. J.: «Intuitionism and Formalism», p. 80.

11 Cf. WITTGENSTEIN, L.: *Über Gewissheit*, edited by G. E. M. Anscombe, with translation by D. Paul and G. E. M. Anscombe: *On Certainty*, B. Blackwell, Oxford, 1969, n. 38/ p. 7 e.

to do Mathematics¹². For him, we construct Mathematics and «fix whether there is to be only one proof of a certain proposition, or two proofs, or many proofs»¹³. Wittgenstein's Mathematics thus assumes an *anthropological* character. This constructivism is different from Brouwer's, because the proof depends on how we prove the proposition. Hence, he is proposing a more radical constructivism than intuitionist constructivism: for Wittgenstein, the calculating of anything depends on whether *one* calculates well or badly¹⁴.

This *constructivism* considers that Mathematics is an engine of creation and not, as the Platonist believes, a device to discover new mathematical facts, and it conceives the proofs as ingredients of Mathematics understood as a creative activity. Wittgenstein is not following any known constructivisms¹⁵: the arithmetization of Mathematics of L. Kronecker; the French semi-intuitionist (J. H. Poincaré and E. Borel); Brouwer's Intuitionism¹⁶; H. Weyl's revision of mathematical practice; T. Skolem's finitism; constructive recursive Mathematics of A. Markov's School; and A. S. Esenin-Vol'pin's ultrafinitism. Wittgenstein position is a *strict finitism*. For A. S. Troelstra and D. van Dalen, the Wittgensteinian conception can share some aspects of the last view insofar as he accepts that Mathematics has no business with an idealized infinite set of natural numbers¹⁷. But his posture is quite original: he connects the distrust of non-constructive proofs with Mathematics as a creative activity. Thus it is possible to accept in Wittgenstein a kind of proof that presents an extension from within (a new concept, a new paradigm, a new insight, ...) and another type of proof that simply adds elements to the mathematical stock, without enhancing our conceptual understanding¹⁸.

Both in his period of transition and in his later thought, Wittgenstein was especially interested in mathematical practice, in the reflection on *mathematical experience*, because «the mathematical proposition has been obtained by series of actions that are in no way different from the actions of the rest of our lives»¹⁹. So, he considers this more important than a detailed analysis of the formal aspects. Kreisel rightly points out that Wittgenstein limits himself to *elementary* mathematical experience, as he

12 Cf. *Bemerkungen...*, V, 5/ *Remarks...*, p. 262.

13 WITTGENSTEIN, L.: *Lectures on the Foundations of Mathematics*, from the notes of R. G. Bosanquet, N. Malcolm, R. Rhees and Y. Smithies, edited by C. Diamond, Harvester Press, Hassocks, 1976, III, p. 39.

14 Cf. *Bemerkungen...*, I, ap. III, 15/ *Remarks...*, pp. 120-121. Cf. GONZÁLEZ, W. J.: «Intuitionistic Mathematics and Wittgenstein», *History and Philosophy of Logic*, v. 12, (1991), pp. 167-183.

15 Cf. TROELSTRA, A. S. and DALEN, D. VAN: *Constructivism in Mathematics*, North-Holland, Amsterdam, 1988, pp. 17-30.

16 Cf. GONZÁLEZ, W. J.: «Is Wittgenstein's Philosophy of Mathematics Intuitionist?», HALLER, R. and BRANDL, J. (ed): *Wittgenstein — Towards a Re-Evaluation*, Hölder-Pichler-Tempsky, Vienna, 1990, pp. 123-128.

17 Cf. TROELSTRA, A. S. and DALEN, D. VAN: *Constructivism in Mathematics*, p. 29.

18 Cf. *Bemerkungen...*, V, 28/ *Remarks...*, p. 285.

19 WITTGENSTEIN, L.: *Über Gewissheit*, n. 651/ *On Certainty*, p. 86.

considers that higher Mathematics can easily divert the attention from the essential²⁰. In my judgement, this is because he considers Mathematics first and foremost an *activity*²¹, and that which is most relevant is found in the least complicated calculations.

2. CHARACTER OF MATHEMATICAL ACTIONS

About the ingredients of the activity called «mathematical practice», we can find firstly that the aim of Wittgenstein's task is not scientific: his purpose is not to construct Mathematics, yet he is able to go deeply into the roots of scientific activity (in this case, mathematical). This is not incoherent, because Mathematics as a subject has no responsibility before Philosophy, but mathematicians use a *language* that could be philosophically confusing, and so they can make mistakes when they develop Mathematics. Secondly, the mathematical experience is understood by him as a sort of *experience*, because Mathematics is an activity in the wide context of activities – forms of life – that correspond to actions of human beings. Both sides explain that, on the one hand, Wittgenstein stresses the philosophical contribution to the clarification of mathematical language, to throw light on actions of mathematicians; and, on the other hand, he emphasizes the severance between the different uses of language (of Mathematics, Psychology, ...), insofar as his aim is descriptive: Philosophy does not change the reality – *Sie läßt alles wie es ist* –²², including human actions.

First of all, human activity that is called «Mathematics» needs a *purpose*, because – as I. R. Shafarevich has pointed out – «any activity devoid of purpose is by this very fact devoid of sense»²³. So, what is the aim of mathematical actions? is a relevant question. Usually the answer is a dicotomy: to make inventions or discoveries. For Wittgenstein, the reply is unequivocal: «the mathematician is an inventor, not a discoverer»²⁴. The discoveries need something to be discovered – mathematical objects – and he rejects such a possibility, adopting a coherent posture because he conceives the mathematical task as a creative activity.

At the bottom of his constructivism, there is an interconnection between inventions and ordinary language: the mathematical development is linked to elemental experien-

20 Cf. KREISEL, G.: «Mathematical Logic: Tool and Object Lesson for Science», *Synthese*, v. 39, (1985), p. 3.

21 Cf. *Bemerkungen ...*, VII, 24/ *Remarks ...*, p. 390. Cf. *Bemerkungen ...*, IV, 32/ *Remarks ...*, p. 241. Cf. *Bemerkungen ...*, V, 15/ *Remarks ...*, p. 273.

22 WITTGENSTEIN, L.: *Philosophische Untersuchungen*, edited by G. E. M. Anscombe and R. Rhees, Blackwell, Oxford, 1953, n. 124; translated by G. E. M. Anscombe: *Philosophical Investigations*, B. Blackwell, Oxford, 1976, p. 49 e.

23 SHAFAREVICH, I. R.: «On Certain Tendencies in the Development of Mathematics», *The Mathematical Intelligencer*, v. 3/4, (1981), p. 183.

24 *Bemerkungen ...*, I, 168/ *Remarks ...*, p. 99.

ce. He considers «essential to Mathematics that its signs are also employed in *mufti*»²⁵. The mathematical actions are meaningful insofar as the signs are used outside Mathematics. But, in this case, it is very difficult that Wittgenstein's focus can go a long way from Applied Mathematics; moreover, in his posture, this Science could be in the situation of being reduced to the most basic aims: to the «purposes of housekeepings», in P. Bernays' expression²⁶. Many times, that is the impression received by the reader. However, C. Wright has pointed out that Wittgenstein «was preoccupied at bottom with the two most fundamental questions to which Pure Mathematics gives arise: the apparent *necessity* of mathematical truths, and the nature of our apparent knowledge of them»²⁷.

His remarks depend heavily on his philosophical task: to describe the *Grammar* of Mathematics and its relation to ordinary language. So, the question «what does Mathematics need a foundation for?» has the following answer: «It no more needs one, I believe, than the propositions about physical objects – or about sense impressions, need an *analysis*. What mathematical propositions do stand in need of is a clarification of their Grammar, just as do those other propositions»²⁸. From this point of view, it is quite reasonable that, for him, the key to an understanding of the «foundations» of Mathematics is clarifying propositions such as 'I must have miscalculated'²⁹.

Among Wittgenstein's ideas, a constant is to clarify the actions in the context of their Grammar, because the sense is there, in connection with the use of language. He accepts that Mathematics rests on propositions, maintaining that they are absolutely fundamental and the point of contact with reality³⁰. For S. Körner, Wittgenstein's views include the following three theses, which he rejects: i) there are no mathematical facts and, hence, no mathematical objects; ii) mathematical propositions are normative (analytical propositions, contrary to their appearance); iii) concepts which occur in mathematical propositions must also occur in non-mathematical propositions. Körner considers the theses mistaken; they may appear plausible so long as one restricts Mathematics to simple, finite Arithmetic³¹.

Undoubtedly, the anthropological character of his Mathematics underlies the three theses; the result is a *strict finitism*. In my judgement, as Körner recognizes, his posture seems plausible for elementary experience. Within this scope, Wittgenstein's conception of Mathematics as activity appears plausibly coherent: it accompanies the lin-

25 *Bemerkungen ...*, V, 2/ *Remarks ...*, p. 257.

26 BERNAYS, P., «Comments on Ludwig Wittgenstein's *Remarks on the Foundations of Mathematics*», *Ratio*, v. 2, (1959); reprinted in BENACERRAF, P. and PUTNAM, H. (eds): *Op. cit.*, p. 522.

27 WRIGHT, C.: *Wittgenstein on the Foundations of Mathematics*, Duckworth, London, 1980, p. VIII.

28 *Bemerkungen...*, VII, 16/ *Remarks...*, p. 378.

29 Cf. *Bemerkungen...*, III, 90/ *Remarks...*, p. 221.

30 Cf. *Lectures...*, XXVIII, p. 267.

31 Cf. KÖRNER, S.: «On Wittgenstein's Conceptions of Logic and Philosophical Grammar», in HALLER, R. and BRAND, J. (eds.), *Wittgenstein – Towards a Re-Evaluation*, Hölder-Pichler-Tempsky, Vienna, 1990.

guistic activity of human beings which have well known forms of life. However, for the purposes of Pure Mathematics it is extremely narrow: it is very difficult to reduce the activity of this Science to something that has no object, is purely normative and requires extramental occurrence to be meaningful. Particularly weak is the last thesis: the History of Mathematics displays mathematical theories which have been developed without specific connection with any empirical phenomena. Other theses («no mathematical objects» and «normativeness») give rise to more controversy in Philosophy of Mathematics, but normally are not peacefully accepted in Wittgensteinian terms.

Secondly, closely linked to the question of mathematical purpose – its sense –, another important problem appears: the *status* of mathematical actions in themselves. In this respect, the interrogant brings about a wide debate: have the mathematical actions a specific nature, quite different from other human actions? It concerns the distinction between the activities of mathematicians and other scientists, mainly of Empirical Sciences. The point has a crucial test in the duality «calculation» – «experiment». Studying the kind of actions in these cases, we can find elements to characterize Mathematics as activity and, consequently, to appreciate the differences with other activities.

For Wittgenstein, the differences between mathematical theories and empirical theories are deep: the sphere of the «mathematical objects» – whatever it may be – and the sphere of empirical facts are quite different. At the same time, in accordance with his general tendency regarding the discourse as split up into a number of distinct islands with no communication between them, the propositions of Natural Science and the propositions of Mathematics are also different³². Therefore, calculation and experiment are different activities: they are members of different thematic «spheres» and distinct «islands» of language.

Again, Wittgenstein's view on Mathematics is original: it is not verificationist – against the «Received view» – and it is not possible to identify with Brouwer's Intuitionism. But he sometimes displays an ambivalent attitude towards Verificationism and Intuitionism. In effect, the empiricist background of Verificationism has a strong attraction for him, but he has a reluctance to accept it³³. The reason for the rejection is the normative character of Mathematics³⁴. So, Wittgenstein is against an empiricist conception of «mathematical fact» and the notion of «proof» as experimental corroboration. For him, we do not require empirical knowledge for the proof: «Mathematics forms a network of norms»³⁵ and also forms concepts. This idea gives the key to the *malpractice*.

32 Cf. DUMMETT, M.: «Wittgenstein's Philosophy of Mathematics», *Philosophical Review*, v. 68, (1959); reprinted in BENACERRAF, P. and PUTNAM, H. (ed): *Op. cit.*, p. 493.

33 Cf. DUMMETT, M.: «Wittgenstein's Philosophy of Mathematics», p. 503.

34 «Mathematics is normative. But 'norm' does not mean the same thing as 'ideal'», WITTGENSTEIN, L.: *Bemerkungen über die Grundlagen der Mathematik*, VII, 61/ *Remarks on the Foundations of Mathematics*, p. 425.

One has calculated rightly or wrongly, when one has been trained in a rule of calculating. The rule has been followed when the result of calculating is normal, i. e., there is agreement in practice. The *calculation* is withdrawn from being checked by experience, but it serves as paradigm for judging experience³⁶. Mathematical propositions have not the function of empirical propositions, however, calculating must be founded on empirical facts – psychological and physiological – that make it possible. Calculating is a *useful activity* in itself: it has its own content and an application to empirical reality³⁷.

From the physiological fact, the calculation gets the status of the picture of an experiment; and from the psychological act, the calculation gets its point, its physiognomy³⁸. Originally, calculation can take place in the medium of imagination³⁹, but this is not to say that calculation – or proof – is fundamentally a private mental activity, as the mathematical intuitionist historically thought⁴⁰. For Wittgenstein, there is a psychical course when we calculate, that is not psychologically investigated; and the important thing is the human being that has mastered the technique of the kinds of calculation (addition, multiplication, ...) ⁴¹. His position is not calculation understood as the result of an introspective construction – a purely mental act –, as is Brouwer's⁴².

Differences with him about how understand this notion arise from the beginning. Wittgenstein adopts an anthropological point of view with special emphasis on training and practice: someone *can* calculate when he shows, to himself as well as to others, that he *calculates* correctly⁴³. He is also far from the empiricist idea of calculation – or proof – as experimental corroboration, because the «consensus belongs to the essence of calculation» and it needs the prediction because in a technique of calculating, «prophecies must be possible»⁴⁴. In short, for him, calculation is an element of a Science that describes successfully the way in which we calculate, and predicts successfully the result which we shall get when we are content that we have calculated properly⁴⁵.

Calculation could sometimes be an experiment, but that *experiment* in itself is not a calculation. It is an experiment when we want to *judge* if someone has mastered the technique of calculation, but in that case the result of the experiment is not a mathemat-

35 WITTGENSTEIN, L.: *Bemerkungen ...*, VII, 67/ *Remarks ...*, p. 431.

36 Cf. *Bemerkungen ...*, VI, 23/ *Remarks ...*, p. 325.

37 Cf. *Bemerkungen...*, VII, 18/ *Remarks...*, p. 382.

38 Cf. *Ibidem*.

39 Cf. *Bemerkungen...*, I, 98/ *Remarks...*, p. 73.

40 Cf. WRIGHT, C.: *Wittgenstein on the Foundations of Mathematics*, p. 333.

41 Cf. *Bemerkungen...*, VII, 18/ *Remarks...*, p. 382; *Bemerkungen...*, VI, 32/ *Remarks...*, pp. 334-335.

42 Cf. «Historical background, Principles and Methods of Intuitionism», p. 141. Cf. *Cambridge Lectures on Intuitionism*, pp. 5-6.

43 Cf. *Bemerkungen...*, VI, 33/ *Remarks...*, p. 335.

44 *Bemerkungen...*, III, 67/ *Remarks...*, p. 193.

45 Cf. WRIGHT, C.: *Wittgenstein on the Foundations of Mathematics*, p. 324.

ical proposition: it is a psychological one. The difference is clear when the teacher makes the pupil do a calculation in order to see whether he *can* calculate; that calculation is an experiment⁴⁶. And Wittgenstein maintains that «if a proof is conceived as an experiment, at any rate the result of the experiment is not what is called the result of the calculation. The result of the calculation is the proposition with which it concludes; the result of the experiment is that from these propositions, by means of these rules, I was led to this proposition»⁴⁷.

Nevertheless, in spite of these differences between *calculation* and *experiment*, he does recognize the possibility of confusion: «the conception of calculation as an experiment tends to strike us as the only *realistic* one»⁴⁸. The mistake is then in the nature of experiment: in believing that whenever we are keen on knowing the end of a process, it is what is called an «experiment». Wittgenstein draws a line between the calculation with its *result* and an experiment with its *outcome*⁴⁹; calculation includes both process and result, but the experiment is independent of its outcome. Furthermore, there is a distinction: in an experiment we have something tangible⁵⁰, whereas calculus is only an ability to do something⁵¹. For him, according to the distinction «calculation»—«experiment», there is a *gap* between Mathematics and Empirical Sciences.

Basically, for Brouwer and Wittgenstein, Mathematics is a *construction*: neither is there an independent mathematical reality waiting to be discovered by us, nor does there exist an independent mathematical reality answering to our mathematical propositions before our constructions. According to Brouwer, each mathematical proposition is *eo ipso* a construction and the proof is always a mathematical construction, so he is against non-constructive proofs: he rejects that a proof establishes the existence of something without providing an effective means of finding it. He defends that every mathematical proposition can be judged: it can be proved or be reduced to absurdity. Furthermore, he maintains that every construction can be attempted only in a finite number of ways and «every assertion of possibility of a construction of a bounded finite character in a finite mathematical system can be judged»⁵².

There is a constructive position with regard to the concept of «proof» in his *Lectures on the Foundations of Mathematics*: finding a proof is constructing a proposition by operating on certain given propositions, called «primitive propositions», according to certain rules. Nevertheless, for Wittgenstein, a) not every construction of a proposition is a proof, because it is possible to construct propositions as ‘It is pitch dark in this

46 Cf. *Bemerkungen...*, III, 67/ *Remarks...*, p. 194.

47 *Bemerkungen...*, I, 162/ *Remarks...*, p. 98.

48 *Bemerkungen...*, III, 76/ *Remarks...*, p. 202.

49 Cf. *Bemerkungen...*, I, 109/ *Remarks...*, pp. 77-78.

50 Cf. *Bemerkungen...*, III, 76/ *Remarks...*, p. 202.

51 Cf. *Bemerkungen...*, III, 81/ *Remarks...*, p. 210.

52 BROUWER, L. E. J.: «Historical background, Principles and Methods of Intuitionism», p. 141. Cf. BROUWER, L. E. J.: *Cambridge Lectures on Intuitionism*, pp. 5-6.

room' according to certain rules, but it would not be constructing a proof; and b) not every proof proceeds from primitive propositions (for example, tautologies). He calls attention to the fact that a proof constructing a proposition is not *eo ipso* a true proposition⁵³. Certainly, as Wittgenstein recognizes, the crucial point is *how* the proof constructs a proposition⁵⁴.

Concerning the *status* of mathematical actions in themselves, as we can appreciate in the notions like «calculation» and «proof», the differences with other Sciences are greater than the similarities (even when «calculation» may be an «experiment»). Brouwer insists in the History of Science: Practical Arithmetic and Geometry «have so far resisted all improvements in the tools of observation» and «until comparatively recent times there has been absolute confidence that no experiment could ever disturb the exactness of the laws of Arithmetic and Geometry»⁵⁵. His own position is clearly Kantian: he accepts *synthetic a priori judgments*⁵⁶.

Meanwhile, Wittgenstein adopts a linguistic posture: he is interested in how we *use* mathematical propositions and, in contrary to Brouwer, he considers unimportant the «inner process» or «mental states» for knowing mathematical actions⁵⁷. For him, language is completely articulated: neither do the analytical propositions have a meaning *sui generis* established only by linguistic operations, nor do the synthetic propositions have a meaning given by some direct association with sense-perceptions that would constitute their verification. He defends that there are differences between propositions of Mathematics and those dealing with empirical facts⁵⁸. In spite of a loss of confidence in the label «the Exact Science», the differences with Empirical Sciences remain: «one cannot contrast mathematical certainty with the relative uncertainty of empirical propositions»⁵⁹.

Thirdly, after the problem of the mathematical purpose and the characterization of its status, the question to resolve affects its level of *autonomy*: does Mathematics as activity depend on «internal factors» or requires the presence of «external factors» to explain its content? Adopting the conception of Mathematics as configured by mental acts – as Brouwer thought – the autonomy has no restrictions, because Mathematics has

53 Cf. WITTGENSTEIN, L.: *Lectures...*, VII, p. 68.

54 Cf. *Bemerkungen...*, III, 29/ *Remarks...*, p. 164.

55 BROUWER, L. E. J.: «Intuitionism and Formalism», pp. 77-78.

56 «The apriority of time does not only qualify the properties of Arithmetic as synthetic a priori judgments, but it does the same for those of Geometry, and not only for elementary two –and three– dimensional Geometry, but for non-euclidean and *n*-dimensional Geometries as well», BROUWER, L. E. J.: «Intuitionism and Formalism», p. 80.

57 Cf. WITTGENSTEIN, L.: *Über Gewissheit*, n. 38/ *On Certainty*, p. 7 e.

58 Cf. WITTGENSTEIN, L.: *Zettel*, edited by G. E. M. Anscombe and G. H. von Wright, B. Blackwell, Oxford, 1967, n. 701; translated by G. E. M. Anscombe: *Zettel*, B. Blackwell, Oxford, 2nd edition, 1967, p. 121 e.

59 *Über Gewissheit*, n. 651/ *On Certainty*, p. 86 e.

an inner architecture⁶⁰. By contrast, following a sociological – or, in general, a intersubjective – point of view, the «external factors» are crucial and the autonomy, in a strict sense, can disappear. Sometimes, Wittgenstein has been directly associated with this second group⁶¹.

Certainly, from a general perspective, in his later writings Wittgenstein put more emphasis on the *intersubjective* elements (the uses, rules, language-games, conventions, ...) than on the other aspects. His clearly pragmatic views highlight those components; nevertheless, the objective ingredients (the extramental reality, the laws of nature, ...) are also taken into account. In my judgement, his notion of «truth» clearly displays the complexity of Wittgenstein's position: *truth* is a certain redundancy which concerns a proposition with sense, expressed linguistically through an asseveration possessing a justification – one or more proofs – and having a conventional character, even when there are present aspects which make it admissible and elements outside the language itself which permit its acceptance⁶². This is the general context for replying to the question about autonomy of the actions of Mathematics: prevalence of intersubjective elements, inclusion of objective ingredients and lack of interest in subjective components.

How to resolve the problem of *agreement* in Mathematics, is a direct way of asking about the mathematical autonomy. An easy – and frequent – answer is to adopt the conventionalism. Is that his line? About this point, there is a conflict of interpretations: the problem is whether he rejects conventionalism or, rather, defends it. On the one hand, R. J. Fogelin – in his reconstruction of Wittgenstein's thought⁶³ – includes the rejection of the *pure conventionalism* by him as one more element. On the other hand, for M. Dummett, «Wittgenstein goes in for a full-blooded conventionalism; for him the logical necessity of any statement is always the *direct* expression of a linguistic convention. That a given statement is necessary consists always in our having expressly decided to treat that very statement as unassailable; it cannot rest on our having adopted certain other conventions which are found to involve our treating so. This account is applied alike to deep theorems and to elementary computations»⁶⁴.

Other authors have drawn attention to the presence of conventionalism in Wittgen-

60 «Intuitionistic Mathematics is inner architecture, and (...) research in foundations of Mathematics is inner inquiry», BROUWER, L. E. J.: «Consciousness, Philosophy, and Mathematics», *Proceedings of 10th International Congress of Philosophy*, Amsterdam, 1940; reprinted in BENACERRAF, P. and PUTNAM, H. (ed): *Op. cit.*, p. 96.

61 Cf. BLOOR, D.: «Wittgenstein and Mannheim on the Sociology of Mathematics», *Studies in History and Philosophy of Science*, v. 4, (1973), pp. 173-191.

62 Cf. GONZÁLEZ, W. J.: «The Notion of Truth in Wittgenstein's Philosophy of Mathematics», in WEINGARTNER, P. and SCHURZ, G. (eds): *Logic, Philosophy of Science and Epistemology*, Hölder-Pichler-Tempsky, Vienna, 1987, pp. 419-423.

63 Cf. FOGELIN, R. J.: «Wittgenstein and Intuitionism», *American Philosophical Quarterly*, v. 5 / 4, (1968), p. 271. Cf. FOGELIN, R. J.: *Wittgenstein*, Routledge and K. Paul, London, 1976, pp. 191-196.

64 DUMMETT, M.: «Wittgenstein's Philosophy of Mathematics», p. 495.

stein, as for example A. Ambrose, who suggests that it is to be found in Wittgenstein's views on the nature of mathematical propositions and, in general, in those on necessary propositions⁶⁵. C. Wright analyses that question in detail and considers it to have been misinterpreted by Dummett: Wittgenstein does not maintain a radical conventionalism. Dummett's interpretation is understandable insofar as Wittgenstein introduces the idea of «decision» into the description of our admission of proof and ratification of necessity, and even into the description of our following the rules⁶⁶. So, in the *Lectures on the Foundations of Mathematics*, Wittgenstein insists on the idea of *decision*: «whether or not we say, 'there must be a mistake in the construction', is a question of decision»⁶⁷.

Now, the key question is how to understand the role of decision-making in the admission of proofs and the application of conventions; because if we accept that Wittgenstein defends a full-blooded conventionalism, then we would accept that he maintains an incoherent position, for it cannot be allowed that what concerns consequences of conventions may be a matter of arbitrary decision. Dummett himself has pointed out that if every one has the right to *lay down* that the assertion of a proposition of a given form is to be regarded as always justified – without regarding the use which has already been given to the words contained in the asserted proposition –, then the «communication would be in constant danger of simply breaking down»⁶⁸.

In order to answer the question on the role of *decision-making* in the admission of proofs and the application of conventions, it is useful to point out the facets that, according to C. Wright, are there in Wittgenstein's position: 1) The function of necessary propositions is normative, and they regulate the use of propositions of which truth or falsity are predicable, so the concept of «truth» is not applicable to necessary propositions. 2) There is no ultimate justification for the principles of inference which we use; there are only considerations of utility appropriate to any conventional practice. 3) Wittgenstein defends that the concepts of «correct inference», «calculation», etc., have no more in them than is defined by the rules which we actually employ. 4) From these considerations, we can conceive the possibility of alternative systems of calculation and inference. 5) We ought not to think of our subscription to particular principles of inference as a rigid contract in understanding, predetermining what in any particular case is to count as their correct application if we are to remain faithful to their content⁶⁹.

The first facet, according to which necessary propositions are not imposed and they function regulating the use of propositions of which truth and falsity are predicable,

65 Cf. AMBROSE, A.: «Mathematical Generality», in AMBROSE, A. and LAZEROVITZ, M. (eds): *Ludwig Wittgenstein: Philosophy and Language*, Allen and Unwin, London, 1972, p. 301.

66 Cf. WRIGHT, C.: *Wittgenstein on the Foundations of Mathematics*, pp. 364-386; especially, p. 375.

67 *Lectures*, V, p. 56. «Whether it is calculating correctly or not depends in a way of your decision», WITTGENSTEIN, L.: *Op. cit.*, XIII, p. 124. Cf. *Ibidem*, II, pp. 30-31 and *Lectures*, XXIII, p. 226.

68 «Wittgenstein's Philosophy of Mathematics», p. 501.

69 Cf. *Wittgenstein on the Foundations of Mathematics*, pp. 375-376.

does not commit him to employing different techniques of inference: it is possible to locate the decision within the framework of the rules which we actually use. In my judgment, this position fits perfectly with Wittgenstein's «linguistic naturalism»⁷⁰, and it is the expression of a conventionalism of a different type from the full-blooded one. The difference between both is clear: in Wittgenstein's conventionalism the decisions about new necessary propositions are not completely arbitrary; whereas in pure or full-blooded conventionalism there is a more radical position, because the decisions are *arbitrary*. So, this element does not fit into Fogelin's reconstruction of Wittgenstein's thought: he was attracted by conventionalism instead of following the opposite way of rejection of conventionalism.

Hence, the autonomy of Mathematics as activity does not disappear in Wittgenstein. The presence of the conventionalist attraction – the relevance of «external factors» – is compatible with preserving the specificity of the actions of mathematicians following rules. Insofar as he asseverates that «the agreement of people in calculation is not an agreement in opinions or convictions»⁷¹, he admits that the agreement in Mathematics has «internal factors». Expressly, he accepts the existence of a relation between *agreement* and *rule*: «the phenomena of agreement and acting according to a rule hang together»⁷². Moreover, the mathematical correctness depends on internal factors and is appreciated in the result⁷³. As N. Malcolm maintains, he holds that the actions «that are in accord with a rule are 'internally' related to the rule, *in the sense* that if you do not do *this* you are not following the rule»⁷⁴.

According to the present analysis, any plausible *characterization* of mathematical actions includes – in my judgement – at least three features: i) the necessity of a purpose which give sense to mathematical work, because the mathematicians are not merely *acting*: they are making inventions or discoveries, which make its activity meaningful; ii) the status of mathematical actions is specific, different from the activities developed in other Sciences, as we can appreciate when comparing *calculation* – or proof – and *experiment*; iii) within the field of the Sciences, Mathematics has a wide autonomy as activity insofar as the «internal factors» have more weight than «external factors», even when the mathematician has the attraction of conventionalism.

70 Cf. GONZALEZ, W. J.: *La Teoría de la Referencia. Strawson y la Filosofía Analítica*, Ediciones Universidad de Salamanca y Publicaciones de la Universidad de Murcia, Salamanca-Murcia, 1986, pp. 60-66.

71 *Bemerkungen ...*, VI, 30/ *Remarks ...*, p. 332.

72 *Bemerkungen ...*, VI, 41/ *Remarks ...*, p. 344. «Acting according to a rule presupposes the recognition of a *uniformity*», *Bemerkungen ...*, VI, 44/ *Remarks ...*, p. 348.

73 «In Mathematics the result is itself a criterion of the correct calculation», *Bemerkungen ...*, VII, 27/ *Remarks ...*, p. 393. «What is proved by a mathematical proof is set up as an internal relation and withdrawn from doubt», *Bemerkungen ...*, VII, 6/ *Remarks ...*, p. 363.

74 MALCOLM, N.: «Wittgenstein on Language and Rules», *Philosophy*, v. 64, (1989), p. 7.

3. THE DESCRIPTION OF MATHEMATICAL ACTIONS

To describe Mathematics as activity is a way of clarifying and distinguishing the components of mathematical actions. This complex task give us an important basis for the explanation of mathematical knowledge and may be an interesting support for designing the course of action of the mathematical developments. Many aspects intervene in this task. Among them, two are especially relevant: the character of mathematical actions – their nature – and the subject-matter. They contribute to configurate the activity that is called «Mathematics», because – as scientific activity – it requires an «object» (the «mathematical objects», whatever it may be) and the method for studying it. Mathematical actions belong to the rational activity engaged in solving-problems within a definite field.

Explicitly, in the last section, the mathematical actions appeared as *specific activity*, irreducible to other scientific actions; and, implicitly, assumed the presence of a *specific subject-matter*, distinct from the rest of the Sciences. Hence, human beings doing Mathematics work a Science that is different from Empirical Sciences: their activity deals with «something» that is not an empirical fact. In my judgment, it has *objectivity*: mathematical actions and, consequently, mathematical communication are possible as scientific activity on the basis of such objectivity. Action Theory may go there, even to contribute to the ontological problem of Mathematics, but its focus only allows an indirect contribution.

Comparing different types of action is often the way selected to clarify a sort of action. It accompanies the main end of the description insofar as emphasizing the analogies and differences can help us when the aim is to shed light on an activity. Frequently, Wittgenstein collects different possibilities in order to look at them through the focus of a comparison. The common element among them is, normally, the idea of *game*. For him, «it is sometimes useful to compare Mathematics to a game and sometimes misleading»⁷⁵. If we compare this Science with the game of chess, the analogy lies in being defined by the rules; meanwhile, there are at least two differences: on the one hand, chess playing as a matter of fact is not used for making *predictions*, whereas Mathematics is used to predict; and, on the other hand, chess has not got an *application*, while Mathematics has an obvious application⁷⁶.

When the comparison between *playing a game* and mathematical actions is used in strict sense, Wittgenstein considers that «the mathematician, in so far as he really is 'playing a game' *does not infer*. For here 'playing' must mean: *acting* in accordance with certain rules. And it would be something outside the mere game for him to infer that he could act in this way according to the general rule»⁷⁷. He also thinks that comparing and acting go hand in hand in a different manner, to discredit certain positions, presu-

75 *Lectures on the Foundations of Mathematics*, XV, p. 142.

76 Cf. *Lectures on the Foundations of Mathematics*, XV, p. 150.

77 *Bemerkungen ...*, V, 1/ *Remarks ...*, p. 257.

mably of B. Russell: «the limits of Empiricism are not assumptions unguaranteed, or intuitively known to be correct: they are ways in which we make comparisons and in which we act»⁷⁸.

Acting according to a rule is at the bottom of his mathematical descriptions, and a feature of a good description is a description *by a rule*⁷⁹. When we describe learning and instruction, what one watches is people following rules, learning to follow rules: an agreement in actions on the part of pupil and teacher⁸⁰. Their following a rule has as a consequence a result, which must be the same at different times, because it is unthinkable that one should follow the rule right and produce different patterns⁸¹. But, as N. Malcolm explains, «a rule, *by itself*, determines nothing»⁸². From this point of view, it is erroneous to interpret Wittgenstein as defending that «the rule and nothing but the rule determines what is correct»: his insight includes the important element of *agreement*. He is maintaining that the rule determines something within a setting of agreement.

Following a rule requires a *consensus of action*: doing the same thing, reacting in the same way. His view of mathematical agreement is supported by an agreement in acting, and not merely in opinions. The basis are in the elementary experience: we all act the *same* way, count the same way. So, «in counting we do not express opinions at all. There is no opinion that 25 follows 24 —nor intuition»⁸³. The rules belongs to a framework of agreement: we *make* a rule «because of the agreement in action —namely, that if we went through these steps we could nearly all get the same results. (...) We get the same result as in the Mathematics books. If we don't get the same, we either (1) find a mistake, or (2) if we don't find the mistake, we say that because of the disagreement, there must be a mistake»⁸⁴.

On the one hand, this description of Mathematics as activity is assuming a regularity in the actions: the agreement in getting a determined result, that is the justification of the calculation; and, on the other hand, a mastery in the use of the rule is required by the mathematicians. For Wittgenstein, it is reasonable to say that «the whole thing is based on the fact that we *don't* all get different results»⁸⁵. In my judgement, he does not dissolve the possibility of objectivity. The mathematical objectivity can appear insofar as it is included in the intersubjective uses of following a rule, as a consequence of the description of the consensus of action: mathematical regularity depends on the regularity in acting and requires the idea of «the same».

78 *Bemerkungen ...*, VII, 21/ *Remarks ...*, p. 387.

79 «To us description *by a rule* is a perfectly good description», WITTGENSTEIN, L.: *Lectures on Philosophical Psychology*, from the notes of P. T. Geach, K. J. Shah, and A. C. Jackson, edited by P. T. Geach, Harvester-Wheatsheaf, London, 1988, p. 28.

80 Cf. *Bemerkungen ...*, VI, 45/ *Remarks ...*, p. 348.

81 Cf. *Bemerkungen ...*, VII, 27/ *Remarks ...*, p. 393.

82 MALCOLM, N.: «Wittgenstein on Language and Rules», p. 9.

83 *Lectures on the Foundations of Mathematics*, XIX, p. 184.

84 *Lectures on the Foundations of Mathematics*, XXXI, pp. 291-292.

85 *Ibidem*, X, p. 102.

If we accept – as H. Wang maintains – that «the words ‘can’, ‘decidable’, etc. mean different things in Pure Mathematics and Applied Mathematics, in actual mathematical activities and the discussions of mathematical logicians»⁸⁶, then we need to confront some difficulties. Firstly, the *scope* of descriptions of mathematical actions, because the differences force us to ask: to what extent they may reach; secondly, the *validity* of the description in itself, insofar as there is the possibility of alteration: can we have alternative descriptions of the same mathematical phenomenon? At the bottom, the great problem is not breaking bridges, recognizing the elements of unity within the differences – semantical, epistemological and methodological – among the distinct parts of this Science (principally, Pure and Applied).

Scope and validity of descriptions are problems that have a direct repercussion in Wittgenstein’s views. His interest is, mainly, in Applied Mathematics and in actual mathematical activities: he wants to describe the real actions of mathematicians from the analysis of their language. But his strict finitism makes it difficult to go deep into Pure Mathematics and, in his later period, he has no will to participate in the discussions of mathematical logicians⁸⁷. The Wittgensteinian position tends to accept that Mathematics *as activity* is sometimes very abstract, but it is not quite independent of the elementary mathematician’s task, even when its content is genuinely abstract. He has a tendency towards considering that the clarification of elementary properties of natural numbers is the key for avoiding new important difficulties in the rest of Mathematics. But, it is very difficult to hold this as a satisfactory explanation of mathematical activity, especially in the light of recent developments of Pure Mathematics (like Ring Theory).

Regardless of these limitations, are Wittgenstein’s descriptions in accordance with real actions of mathematicians? His descriptions are clearly anthropological, as we can appreciate clearly in the remarks on calculus⁸⁸. From the same phenomenon, it is possible to offer an alternative description in other terms: that is the case of I. Lakatos’s works on Mathematics. About his careful descriptions in *Proof and Refutations*⁸⁹. M. Dummett has written that «they have the merit of really having been based on seeing what we actually do, as, despite his advocacy of that way of proceeding, Wittgenstein’s do not»⁹⁰. At the same time, he is not recommending Lakatos’s philosophical conclusions more than Wittgenstein’s.

86 WANG, H.: «Theory and Practice in Mathematics», in WANG, H.: *From Mathematics to Philosophy*, Humanities Press, N. York, 1974; reprinted in TYMOCZKO, T. (ed): *New Directions in the Philosophy of Mathematics*, Birkhäuser, Boston, 1985, p. 137.

87 Cf. *Bemerkungen ...*, V, 24/ *Remarks ...*, pp. 281-282.

88 «The calculus makes no predictions, but by means of it you can make predictions», *Lectures on the Foundations of Mathematics*, XV, p. 150.

89 Cf. LAKATOS, I.: *Proof and Refutations. The Logic of Mathematical Discovery*, edited by J. Worrall and E. Zahar, Cambridge University Press, Cambridge, 1976 (reprinted with corrections in 1979).

90 DUMMETT, M.: «Reckonings: Wittgenstein on Mathematics», *Encounter*, v. 50/ 3, (1978); in SHANKER, S. (ed): *Ludwig Wittgenstein: Critical Assessments*, v. 3, Croom Helm, London, 1986, p. 119.

Lakatos's descriptions are less anthropological, more historical, and with greater methodological content than Wittgenstein's. Their level of validity is higher than the latter's, insofar as he tries to collect the elements of a typical pattern of mathematical discovery: the activity that – in his view – involves conjecture, proof, counterexamples to the conjecture, and re-examinations of the proof. His description of the formation of mathematical knowledge includes the dynamic components of the mathematical actions. This historical emphasis contributes more to actual Mathematics than does the view based on elementary mathematical experience of following a rule. Nevertheless, the inconveniences do not disappear: the examples are very limited; he does not develop the possible counterexamples; and the philosophical consequences of his descriptions require more study.

Originality is the main trait of Wittgenstein's descriptions on mathematical actions. They belong to an interpretation of Mathematics as activity which is completely new: a creative activity in accordance with the actual use of language. He wants to clarify and distinguish the components of the activity of following a rule by the mathematicians, which agree on the basis of a consensus of action. At the same time, according to the present analysis, it is precisely there where the maximum deficiency has its roots: a restrictive conception of mathematical actions, based on elementary Mathematics, which he wants to generalize to this Science as a whole, with sensitive difficulties to shape the most abstract aspects of the activity of Pure Mathematics.

Describing mathematical actions, he avoids Brouwer's mentalism (and, consequently, the introspective character of this Science), without rejecting the role of mental acts in mathematical activity, in the configuration of mathematician's task. But the characterization of what the mathematicians actually are doing is closer to other attempts, like Lakatos's analysis about proof and refutations, than to the remarks of his writings. In my judgement, it is possible to retain Wittgenstein's sharp comments on mathematician's activities, such as that concerning the distinction between «experiment»—«calculation»; nevertheless, the necessity of a more accurate characterization of this activity and its philosophical consequences remains clearly open. We require a different theoretical framework for explaining Mathematics as activity, one which can go beyond his emphasis on the finitude and the situational character of human experience, and which collects the actual progress of this Science.

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