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Automaticity in subtractions depends on problem-size

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Título: La automaticidad en las restas depende del tamaño del problema.
Resumen: Existe considerable evidencia que muestra que las multiplicaciones y las sumas simples se resuelven de manera directa y automática. Sin embargo, la evidencia sobre la automaticidad de restas y divisiones es menos convincente. Usando el paradigma de interferencia en la operación, el presente estudio explora si el resultado de una resta puede ser recuperado inintencionadamente y el rol que juega el tamaño del problema en este proceso. Sesenta y dos participantes tomaron parte en este estudio y tenían que decidir si el resultado de una adición era o no correcto. En las adiciones incorrectas el resultado podía ser la sustracción de los sumandos (7 + 4 = 3) o un número no relacionado (7 + 4 = 5). Nuestros resultados mostraron más errores y respuestas más lentas en aquellos problemas cuyo resultado era la sustracción de los sumandos que en los problemas no relacionados. Sin embargo, estos resultados sólo se encontraron en problemas pequeños (7 + 4 = 3 vs. 7 + 4 = 5) y no en problemas más grandes (14 + 8 = 6 vs. 14 + 8 = 7). Estos resultados sugieren que las sustracciones pequeñas pueden ser recuperadas directamente, cuestionando la existencia de disociaciones entre operaciones. Argumentamos que dependiendo de nuestra experiencia, las mismas representaciones y procesos pueden estar implicados en la resolución de multiplicaciones, adiciones y sustracciones.
Palabras clave: Sustracciones; automaticidad; tamaño del problema; resolución aritmética.

Abstract: The evidence showing that simple multiplications and additions can be solved by direct retrieval is considerable. However evidence about division and subtraction is less compelling. By using a "cross-operation interference paradigm" the present research explores whether subtraction problems can be retrieved without intention and the role of operands' problem-size in this process. Sixty-two participants decided whether the displayed addition was correct or not. In "false additions problems" the answer could be the result of the subtractions of the addends (e.g., 7 + 4 = 3) or an unrelated number (e.g., 7 + 4 = 5). Results showed an interference effect, that is, more errors and slower response times in subtraction related problems than in unrelated problems. More importantly, this effect was restricted to small problems (7 + 4 = 3 vs. 7 + 4 = 5), whereas no differences were found for large problems (14 + 8 = 6 vs. 14 + 8 = 7). These results suggest that small subtractions can be retrieved directly as multiplications, questioning a traditional dissociation between operations. We argue that, depending on individual experience, the same representation and processes can be involved in solving additions, subtractions and multiplications.
Key words: Subtractions; automaticity; problem-size; arithmetic problem solving

Introduction

Simple arithmetic problem solving is so commonplace that frequently we are not aware of its relevance and complexity. In fact, without this ability it would be risky to go shopping, it would be difficult to understand our watches or to know the influence of scoring a goal on the result of a soccer match. Fortunately for most of us, we can solve these kinds of problems easily. Cognitive psychologists have been trying for several decades to understand how our mathematical skills work, but in spite of this, the processes involved in solving arithmetical problems remain the subject of an intense debate (e.g., see Campbell, 2005, for review).

Models of mathematical cognition make different assumptions regarding how simple arithmetic problems are solved. According to the Triple-Code Model (Dehaene, 1992; Dehaene & Cohen, 1995) additions and multiplications are retrieved directly from memory, because verbal sequences such as /*three times six eighteen*/, which were overlearned during schooling, are stored in our memory. According to this mechanism the processing of the operands would trigger the completion of the verbal sequence using rote verbal memory. On the other hand, when participants have to solve subtractions, the operands are processed as quantity representations and the result is obtained by using semantically meaningful manipulations (e.g., counting). This procedure is followed "…whenever rote verbal knowledge of the operation result is lacking, most typically for subtraction problems" (Dehaene & Cohen, 1997, p.322). Although Dehaene, Piazza, Pinel & Cohen (2003) have attenuated this view suggesting that some subtractions could be recovered automatically, empirical evidence was not shown. In a similar vein, other models of mathematical cognition (McCloskey, 1992; Cipolotti & Butterworth, 1995) consider that arithmetic operations are solved with specific representations and mechanisms that are fully independent. Consequently, subtraction would follow different procedures and would employ different mechanisms to those employed in multiplication solving.

Contrasting with the above models, some theories about arithmetical fact retrieval consider that repeated encounters with multiplications and additions is in the origin of the development of strong associations between problems and their corresponding solutions (e.g., Ashcraft, 1987, 1992; Cambpell, 1987; Siegler & Jenkins, 1989). In other words, problem frequency would influence if a problem is solved by procedural or automatic retrieval. Although these models have been applied to multiplications, the learning mechanism they propose could be applied to any operation. Hence, the more frequent subtractions might be learnt for associations and thus, they could be solved by direct retrieval just as is the case of multiplications and additions.

Research has shown that simple additions (e.g., 2 + 3; LeFevre, Bisanz, & Mrkonjic, 1988; Winkelman & Schmidt, 1974; Zbrodoff & Logan, 1986) and multiplications (e.g., 2 x 3; Galfano, Rusconi & Umiltà, 2003; García-Orza, Damas-

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López, Matas & Rodríguez, 2009; Thibodeau, Lefevre & Bisanz, 1996; Zbrodoff & Logan, 1986), are usually solved by retrieval. This memory retrieval process is considered to be fast, accurate, does not require much attentional resources (Campbell & Alberts, 2010; García-Orza et al., 2009; Imbo & Vandierendonck, 2008) and is usually associated with a high level of mathematic skills (e.g., Campbell & Xue, 2001; LeFevre et al., 1996). In other words, retrieval of solutions seems to be automatic. Interestingly, the automatic retrieval of multiplications and additions seems to be modulated by the problem-size effect (problems with smaller operands are solved faster) as retrieval seems to be the preferred strategy for small problems (e.g., 3 x 4, 3 + 4), while different strategies would be employed to large problems (e.g., 8 x 9, 8 + 9; see Zbrodoff & Logan, 2005, for review). The strongest support for this statement comes from self-report studies: participants report the use of automatic retrieval for small additions and an increment in the use of procedural strategies, like repeated addition or combination of operations, for large problems (see e.g., Hecht, 1999; LeFevre et al., 1996). These results seem to support the aforementioned models of mathematical cognition.

However, the evidence for the use of automatic retrieval in simple subtractions, the focus of our interest, is less convincing and more disputed. LeFevre, DeStefano, Penner-Wilger and Daley (2006) conducted a study in which their participants had to solve subtractions and inform how they had solved each problem. This research showed that a large percentage of their participants used retrieval to solve small subtractions (i.e., minuend up to 10) most of the time (see Geary, Frensch & Wiley, 1993; Seyler, Kirk & Ashcraft, 2003; Campbell & Xue, 2001, for similar results). Interestingly, when large subtractions (i.e., minuend between 11 and 18) were presented, participants reported that they tended to use an inverse-reference strategy. The inverse-reference strategy consists in using the inverse arithmetical fact to solve operations, so in the case of the subtraction 13 – 6 = 7, participants may adopt the inverse addition, 7 + 6 = 13, to solve the operation. Although verbal reports are not free from criticism (see Kirk & Ashcraft, 2001), these data suggested that retrieval was employed by participants mainly in relation to small problems (97-73%) but also in large subtractions (66-42%). More recently, Campbell and Alberts (2010; Campbell, 2008) used an inverse format paradigm to explore the use of the inverse fact strategy for solving subtraction. Under this paradigm, both the standard subtraction format (13 – 6 =?) and the inverse subtraction format (6 + ? = 13) were presented. These authors, based on the assumption that participants may use the inverse-reference paradigm in large subtractions, argued that when the inverse subtraction format is presented, participants should be able to speed-up their response, especially in the case of large subtractions. Accordingly, they demonstrated that large subtractions problems (minuend > 10) were solved more quickly when the operation was presented in inverse format. These results were not obtained in the case of small subtractions (minuend < 10; see for similar results in divisions Mauro, LeFevre & Morris, 2003). Campbell and Alberts' results seem to indicate that different strategies are used to solve small and large subtractions, and, although they suggest that retrieval would be the preferred strategy for small problems, their data do not clarify whether automatic retrieval or counting was used.

Thevenot, Castel, Fanget and Fayol (2010), using the operand recognition paradigm (see Thevenot, Fanget & Fayol, 2007) also found evidence indicating that both high and low skilled arithmetic performers solve small subtractions (minuend < 10) by memory retrieval. Moreover, they found that highly skilled participants also solved subtractions involving minuend from 11 to 18 by retrieving their results directly from long term memory. Although this paradigm has been brought into question (see Metcalfe & Campbell, 2010, 2011), it suggests that not only small subtractions, but also large subtractions, could be recovered directly from memory. However, these same researchers have recently questioned the use of retrieval strategies in relation to addition and subtraction. Fayol and Thevenot (2012) found that participants solved one-digit additions and subtractions faster when the operands were preceded by their sign (+ or -) than when the problem was presented simultaneously. On the contrary, no differences were found in multiplications. They concluded that different procedures were employed for multiplications —memory retrieval— than for additions and subtractions —compacted procedures presumably based on quantity representations—.

Clear evidence supporting that subtractions can be automatically retrieved from long term memory comes from a study by Lara-Carmona, García-Orza and Carratalá (2009) using a "cross-operation-interference paradigm". This paradigm has provided significant support for the automatic retrieval of multiplication facts, as it has shown that people are slower in rejecting false additions when the stated result is the correct result of a multiplication (associative lure; e.g., 3 + 4 = 12) (see, e.g., Winkelman & Schmidt, 1974; Zbrodoff & Logan, 1986). That is, participants' response times are slower when the displayed (incorrect) result is the correct result of another operation, a multiplication in this case, and this suggests that the solving of the sums is being interfered by multiplications. Using this paradigm Lara-Carmona et al. (2009) presented false additions that could be the result of a correct subtraction (e.g.: 5 + 2 = 3) or unrelated errors (e.g., 5+2= 4). Participants took longer to recognize as an error those false additions that were subtraction-related as opposed to those unrelated. This means that a problem like 5 - 2 = 3 was also represented in participants' minds, and this interfered with the operation displayed, slowing down its rejection. Unfortunately, larger subtractions (minuend > 10) were not tested in their study, hence it is not clear whether the same pattern of results would be found when using such stimuli.

The purpose of the present research is to investigate whether subtractions are retrieved automatically. Our hypothesis is that the use of direct retrieval strategy does not depend on the type of operation, as is usually assumed (e.g., Dehaene, 1992; Dehaene & Cohen, 1997; McCloskey, 1992) but depends instead on previous experience with those problems (frequency). According to the literature, problem-size can be considered to be an indirect measure of frequency: Ashcraft and Christy (1995) reported lower frequencies of larger multiplication problems in arithmetic books for primary schools. Additionally, in daily life it is reasonable for us to deal with small problems more frequently (Dehaene & Mehler, 1992). Hence, it seems that large problems could be less well stored in our memory than small problems as consequence of their more reduced frequency. This argument applied to subtractions implies that small subtractions should be more strongly associated (with their solution) in our memory than large subtractions and, hence, small subtractions could be retrieved directly from memory, as it is the case in additions and multiplications.

In the research presented here we have tried to determine whether small (one-digit minuend minus one-digit subtrahend) and large (minuend between 11 and 17 minus one-digit subtrahend) subtractions are recovered automatically. With this aim, we have used a cross-operation interference paradigm where participants had to decide whether the sum presented was correct (verification task). Incorrect results could be the correct result of a subtraction (interference condition, e.g., 7 + 3 = 4) or an unrelated number (control condition, e.g., 7+3=5). If both small and large subtractions are retrieved directly from memory, participants' longer response times will be longer under subtraction-related condition than in the control condition. If only small problems are solved by retrieval an interaction between problem-size and type of problem is expected (e.g., Thevenot et al., 2010). Finally, no effects are expected if, as assumed by some authors (e.g., Cohen & Dehaene, 1997; Dehaene, 1992; Fayol & Thevenot, 2012), subtractions are solved simply by using a non-retrieval mechanisms, i.e., procedural strategies, that involve the use of the quantity codes (semantic elaboration).

Method

Participants

Sixty-two undergraduate psychology students took part in this experiment for course requirement (age range: 21-43, 54 females). They were naïve regarding the purpose of the study. They had normal or corrected to normal vision and reported no deficits in numerical or calculation skills. They all had attended to Spanish schools where only multiplication tables are taught by verbal recitation.

Apparatus

Stimulus presentation and data recording were accomplished via ERTS software (ERTS: Beringer, 1999) under MS-DOS, running on a Pentium 133 PC. The stimuli were presented on a 17" color monitor, with refresh cycles of 60 msec.

Materials

Stimuli employed in the experiment appear in Appendix 1. We selected 20 sums for the experiment. Half of the operations were large problems (i.e. 14 + 8), where the first addend was a number between 11 and 17 and the second addend a one-digit number. The other half of the problems were small ones (i.e. 7 + 3), where both addends were one-digit numbers. In the correct response condition, each sum was followed by its correct result (i.e. 14 + 8 = 22). In the incorrect response condition the same sums were employed, but this time each sum was followed either by the correct result of the subtraction of the addends (i.e. 14 + 8 = 6; subtraction-related condition), or by a number without any relation with the addends (i.e.: 14 + 8 = 7); unrelated condition). The solutions provided for false problems were always a one-digit number. To avoid the influence of split effects (i.e., in incorrect problems, the bigger the distance of the incorrect result to the correct result, the faster its rejection, see Ashcraft & Battaglia, 1978), the numerical distance between the results in the different conditions and in the correct results was the same.

A standard cross-operation-interference-paradigm was used (see Lefevre et al., 1988): stimuli were presented in two identical blocks. In each block, the 20 sums were presented followed by a correct result twice. In the incorrect results, the 20 sums were presented once in the interference condition and once in the unrelated condition. This makes a total of 80 trials per block making a total of 160 trials for each participant during the whole experiment. This counterbalancing assures that the results will not be due to specific features of the different numbers involved in each operation[1].[2]

Procedure

For the experimental task, participants sat in front of a computer monitor placed at an approximate distance of 60 cm. Stimuli were presented on a 17-in. color screen in a white on black background with Arial font 24. A trial began with a fixation point (#) during 500 ms and was followed immediately by the problem and its solution which remained

[1] Although the standard cross-operation-interference-paradigm comprises two different blocks (see Lefevre et al., 1988), one of the reviewer suggested that it might be possible that the continuous repetition of problems through the experiment might induce an artificial activation of arithmetic facts. To test this hypothesis, a three-way ANOVA was performed on median correct response times, with relationship (subtraction related vs. unrelated), problem-size (small problem-size vs. large problem-size) and block (one vs. two) as within participants factors. We found a main effect of problem size [$F(1, 61) = 56.3$, $p < .01$] and an interaction between relationship and problem size [$F(1, 61) = 9.73$, $p < .01$]. Importantly, neither a main of effect of block nor an interaction involving this variable arose (all Fs < 1). This seems to indicate that the repetition of the problems in the experiment did not conduct to the artificial activation of arithmetical facts.

in the center of the screen until participants' response. Participants were instructed to indicate as rapidly and as accurately as possible if the result presented with the sum was correct or incorrect by pressing the right or left button respectively. The order of trials was randomized for each participant.

Design

Our interest is only focused on "no" responses or rejections, as only those results allow us to explore our hypothesis about automaticity in subtractions. The variables manipulated were problem-size (small vs. large) and relationship (subtraction-related, unrelated). This resulted in a within subjects 2x2 factorial design.

Results

Two two-way ANOVAs were performed on median correct response times and mean errors, with relationship (subtraction related vs. unrelated) and problem-size (small problem-size vs. large problem-size) as within participants factors. Median was used in responses time to avoid extremes values. Median latencies and percentage of error responses are shown in table 1.

Table 1. Mean response times in milliseconds and mean proportion of errors for correct problems and incorrect subtraction-related and unrelated addition problems in small and large problems (Standard Deviation between brackets).

	Correct Problems		Incorrect Problems			
			Subtraction Related		Unrelated	
	RTs	Errors	RTs	Errors	RTs	Errors
Small problems	941 (199)	.03 (.02)	1045 (203)	.08 (.07)	1012 (177)	.03 (.03)
Large problems	1032 (204)	.04 (.03)	1131 (208)	.04 (.04)	1154 (239)	.04 (.04)

Latency analysis included only correct responses times to incorrect problems (i.e. Correct rejections). The ANOVA revealed a non-significant effect of relationship, $F(1, 610) = .25$, $MSE = 6679.1$; $p = .61$, $\eta^2 = .004$. A significant effect of problem-size was found, $F(1, 61) = 54.2$, $MSE = 15022.2$; $p < .01$, $\eta^2 = .047$, showing that rejecting large problems took 115 ms more than rejecting small problems. More interestingly, an interaction between relationship and problem-size was found, $F(1, 61) = 7.6$, $MSE = 6404.6$; $p < .01$, $\eta^2 = .11$. This interaction showed that the difference of about 33 ms between the subtraction-related and unrelated conditions in small problems was significant, $t(61)=2.88$; $p < .01$. Regarding large problems, in which even the unrelated problems were rejected 23 ms slower than subtraction-related (an inverse interference effect), no significant differences arose, $t(61)= -1.33$; $p = .18$.

Error rates were very low (4.2%). The two-way ANOVA on mean error responses revealed a significant main effect of relationship, $F(1, 61)=19.6$, $MSE=.002$; $p < .01$, $\eta 2= .24$, showing about 2.5 % more errors in the subtraction-related condition than in the unrelated condition. No effects of problem-size were found, $F(1, 61)=2.9$, $MSE=.002$; $p = .93$, $\eta 2= .004$. As in the analysis of response times, an interaction effect between relationship and problem-size arose, $F(1, 61) = 21.4$, $MSE = .002$; $p < .01$, $\eta^2 = .25$, showing differences between subtraction-related and unrelated conditions in small problems, $t(61)= 5.38$, $p < .01$, but not in large problems, $t(61)= 0.05$, $p = .61$.[2]

No accuracy trade-off was observed as indicated by a positive, although non-significant, Spearman correlation between median RT and mean proportion of errors over the four cells of the design, $r=+.31$, $n=4$, $p = .68$.

General discussion

The present research was aimed to investigate whether subtractions are retrieved directly from our memory and whether this retrieval is modulated by problem-size. To accomplish this we used a cross-operation interference paradigm in a verification task. Participants had to indicate whether the operation presented was correct or not. Results showed that participants took longer to reject subtraction-related additions (e.g. 7 + 5 = 2) than unrelated additions (e.g. 7 + 5 = 3), suggesting that the representation of the corresponding subtraction (e.g., 7 – 5 = 2) was activated in their memory.

[2] As one reviewer indicated, in our experiment, the variable problem size has two levels: one digit problems and two digits problems. So it might be possible that our results are due to different strategies used for one and two digit problems rather than to the problem size. An additional analysis was conducted to rule out this hypothesis. We use a similar procedure to Campbell and Xue (2001) to categorize problems as small, medium or large: a problem was considered small if the result of the multiplication of its addends was less than 25 (seven operations). A problem was considered medium if the result of the multiplication of its addends was between 25 and 70 (six operations). Finally, a problem was considered large if the result of the multiplication of its addends was more than 70 (seven operations). A two-way ANOVA was performed on median correct response times and on mean errors with relationship (subtraction related vs. unrelated) and problem-size (small problem-size vs. medium problem-size vs. large problem-size). As the results were similar to those reported in the result section, only the results involving the interaction between relationship and problem size will be reported. For reaction times, an interaction between problem-size and relationship arose [$F(2,122)=4.4$, $p < .02$]. To further explore this interaction, multiple comparisons (Bonferroni corrected) were performed. For small problems subtractions related problems were rejected slower than unrelated problems ($p < .01$). However, these differences were not found either in medium or large problems (both ps>.05). For mean errors, the interaction was also significant [$F(2,122)=11.4$, $p < .01$]. Multiple comparisons revealed more errors in subtractions related problems in both small and medium problems (both $ps < .01$) than in unrelated problems. However, these differences were not found in large problems ($p > .05$). All together, these results seem to show that the problem-size, and not the presence of one or two digit in the addends, modulates the way we solve subtractions.

However this effect was modulated by the problem-size: interference was only obtained for small problems (one-digit addends: e.g., 7 + 5 = 2) but not for large problems (first addend between 11 and 17, e.g., 17 + 8 = 9). These data seem to indicate that small subtractions are recovered directly from our memory whereas no evidence of such activation is observed in larger problems.

Alternatively, it is possible that when presented with 7 + 3 = 4 participants activate the fact that 7 = 3 + 4, in this case, the true addition relationship among the elements of the problem would interfere with performance, questioning the direct retrieval of subtraction problems. Although this alternative account of our data cannot be fully discarded, it is difficult to assume for several reasons: i) It implies that when presented with an addition problem, e.g., 7 + 3 = 4, participants activate all the different combinations of additions between the three numbers, i.e., (7 + 3; 7 + 4 and 3 + 4) what seems implausible. ii) Campbell and Albert (2010) found that when solving big subtractions (minuend > 10) participants sometimes rely on an addition-based strategy, however, no evidence of the use of this strategy was found when presented with small subtractions. This is in line with our account of an automatic solving of these problems. iii) Additionally, it seems also implausible that the strategy of solving subtraction problems by reversing the direction of the problem and then solving an addition could be employed by participants when, in this case, it has a detrimental effect on the task.

Our results extend previous findings regarding the use of retrieval in subtraction problems that had been obtained in self-report studies (e.g., Cambpell & Xue, 2001; LeFevre et al., 2006; Seyler et al., 2003) as well as in experiments that employed other paradigms (Thevenot et al., 2010). Our results directly replicate those by Lara-Carmona et al., (2009) which, using exactly the same task as was employed in this experiment, found a cross-operation interference effect of about 65 ms in small problems. The interference effect found in the current experiment by virtue of these problems (33 ms) provides additional support to that data, but also extends it by demonstrating the lack of such effects in the case of large subtraction problems (those with a minuend between 11 and 17). In addition, the problem-size effect found in this experiment agrees with the strong discontinuity found both in terms of response times and errors described by Seyler et al. (2003) when the size of the minuend in subtraction problems was larger than 10.

The findings in the present experiment have important implications for number processing accounts. The triple-code model (Dehaene, 1992; Dehaene & Cohen, 1995) and other models of arithmetical cognition (McCloskey, 1992; see also Cipolotti & Butterworth, 1995) assume that subtractions are solved through procedures, like counting, that involve the use of the magnitude code. No direct retrieval is expected to be found in these problems and if it is then it would be residual. On the contrary, the present research supports the hypothesis that small (the most frequent) subtractions, such as 7 - 3 are retrieved directly from our memory. The evidence has been obtained using a cross-operation interference paradigm, i.e., a paradigm usually considered as a marker of automatic retrieval (e.g., Winkelman & Schmidt, 1974; Zbrodoff & Logan, 1986). According to our findings, the use of memory retrieval or strategic procedures would not depend on the type of operation presented (i.e., addition, multiplication or subtraction) but on the experience previously gained in regarding those problems. The same mechanism proposed by models of arithmetic fact retrieval to generate associations between multiplication problems and their solution, frequency (see Ashcraft, 1987, 1992; Cambpell, 1987; Siegler & Jenkins, 1989), would also be responsible in the case of subtractions. Small problems seem to be more frequent in our daily life (Ashcraft & Christy, 1995; Dehaene & Mehler, 1992), being exposed to them repeatedly makes the establishment of associative links between the operands and the solution in our memory possible. These associative links allow us retrieving the information directly. On the contrary, as large subtractions are less frequent, we have not developed strong enough associations between the operands and their corresponding solution in our memory. This impedes an automatic access to the solution from the operands, and would force people to rely on the use of procedural strategies that would involve the use of semantic codes (e.g., inverse addition or counting) to solve large problems. If as suggested by a classical model of arithmetic, Siegler's distribution of associations model (e.g., Siegler & Jenkins, 1989), individuals generate an association between problems and the way they should be solved, our results suggest that small subtractions, at least for university students, are associated with a memory retrieval procedure, while large subtractions would be associated with strategic procedures.

Despite the conflict between our data and the proposal of most of the arithmetical cognition models (e.g., Cipolotti & Butterworth, 1995; Dehaene, 1992; Dehaene & Cohen, 1995; McCloskey, 1992), our results and their theoretical consequences can be easily accommodated by them. Simply, it should be assumed that experience (and size would be an indirect measure of frequency) would play an important role in the way people solve arithmetic problems, being the type of operation less relevant. No specific representations have to be proposed for each operation, instead, operations seems to be solved by two different procedures that would imply different mechanisms and representations and, consequently, cerebral structures. One way of solving problems would imply semantic elaboration, this procedure would include different types of strategies that will have in common the manipulation of magnitude representations, and would be more demanding in terms of working memory resources. The other way of solving problems would be through a memory retrieval process that would depend on the strength of associations between the operands and its solution. Using one or other procedure would not depend on the type of operation, but on the educational strategies and the frequency of occurrences of each arithmetical problem individuals are exposed to. If a solution is available by retrieval with a sufficient level of confidence, this procedure would be employed, otherwise the individual would call on semantic elaboration.

Working on the assumption that retrieval is also employed in subtraction, a look at those self-report studies that have compared the use of strategies in the three basic operations reveals that multiplication is the operation more usually solved by retrieval, followed by additions and finally by subtractions and divisions (see e.g., Campbell & Xue, 2001).

Conclusion

The present research adds weight to other experimental evidence that suggests that small subtractions can be solved by retrieval. These results do not rule out the possibility of using different procedures and representations to solve the four basic operations. We simply argue that although different processes (e.g., semantic elaboration and memory retrieval) and representations (e.g., abstract numerical representations-verbal associations) can be employed to solve different operations, the experience previously gained with each particular problem seems to modulate the use of the one or the other procedure. Within this frame, the frequency of solving seems to play an important role in the direct retrieval of the result of the problem, irrespectively of the type of operation. This possibility has been neglected by most of the relevant models of arithmetical cognition that assume, in a simplistic way, that different procedures are involved in solving multiplications and subtractions. The evidence provided here suggests that small subtractions are usually solved by retrieval.

Further research should be devoted to testing subtraction retrieval in people with different arithmetical skills, as this could provide converging evidence about the role of experience in the way arithmetical operations are represented in our mind. As suggested by Thevenot et al. (2010), participants with different skills should show a different pattern of behavior mainly with larger operations, these problems could be solved by memory retrieval by some highly skilled participants but not by low-skilled.

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Appendix I

Stimuli employed in the verification task. Addition problems were presented twice with the corresponding correct result and once with each of the two incorrect result conditions: subtraction- related results (i.e., the result is correct if the arithmetic problem was a subtraction) and unrelated results.

Addition Problems	Correct Solutions	Subtraction- Related Solutions	Unrelated Solutions
Small problems			
7+3=	10	4	5
9+2=	11	7	6
8+1=	9	7	6
9+4=	13	5	6
5+1=	6	4	3
6+2=	8	4	5
8+6=	14	2	1
7+4=	11	3	2
6+4=	10	2	3
5+2=	7	3	4
Large Problems			
11+3=	14	8	6
11+6=	17	5	8
12+8=	20	4	8
13+6=	19	7	9
14+5=	19	9	5
14+8=	22	6	7
15+7=	22	8	9
16+7=	23	9	6
16+9=	25	7	4
17+8=	25	9	9