

Cooperative learning in mathematics: a study on the effects of the parameter of equality on academic performance

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Título: Aprendizaje cooperativo en matemáticas: Un estudio de los efectos del parámetro de igualdad sobre el rendimiento académico.

Resumen: El principal objetivo de esta investigación es determinar la importancia del contenido de aprendizaje y el papel de los conocimientos previos de los alumnos para la formación de los grupos de aprendizaje cooperativo. La investigación se desarrolló en tres aulas de matemáticas de un centro de Educación Secundaria y la muestra estaba formada por 72 alumnos de 3º curso de ESO. Los resultados vienen a demostrar la existencia de una correlación negativa entre el parámetro de igualdad (cooperación, colaboración y tutoría) y la mayor o menor proximidad cognitiva existente entre los conocimientos previos de los alumnos y la estructura de los contenidos de aprendizaje.

Palabras clave: aprendizaje cooperativo; parámetro de igualdad; enseñanza de las matemáticas; conocimientos previos; educación secundaria.

Abstract: The aim of this research is to determine, the importance of learning content and the role of students' prior knowledge for the formation of cooperative learning groups. The research was conducted in three mathematics classrooms at a secondary school and the sample was composed of 72 third year students. The results prove the existence of a negative correlation between the equality parameter (cooperation, collaboration and peer-tutoring) and the degree of existing cognitive proximity between students' prior knowledge and structure of the learning content.

Key words: cooperative learning; equality parameter; mathematics teaching; prior knowledge; secondary education.

Introduction

Ever since two meta-analyses carried out during the eighties by the Minnesota School proved conclusively the superiority of cooperative learning over the competitive and individualist ones (Johnson & Johnson, 1987; Johnson, Maruyama, Johnson, Nelson, & Skon, 1981), the use of said methodology as a backup for education has been increasing exponentially along the last three decades (Garfield, 2013; Nunnery, Chappell, & Arnold, 2013; Slavin, 2011). These results have remained the same for every area of the educational curriculum, and very especially, for mathematics (Cheung & Slavin, 2013; Hossain & Tarmizi, 2013; Lehrer & Lesh, 2013; Plass et al., 2013; Pons, González-Herrero, & Serrano, 2008; Slavin & Lake, 2009; Suri, 2010), being this particular area of knowledge one of the most prolific in both manuals and compilations published (Davidson, 1990a; Sities & Bueth, 2010; Strebe, 2010), as well as in the different research carried out, something which has resulted in numerous research reports and hundreds of scientific and informative publications (Eisenhauer, 2007; Kagan & Kagan, 2005; Slavin et al., 2013).

Likewise, the rise of cooperative learning to this discipline has brought about not only the implementation of general methods of cooperative learning in this area of knowledge, such as Jigsaw (Naomi & Githua, 2013; Novianti, 2013a; Zakaria, Solfitri, Daud, & Abidin, 2013), STAD (Novianti, 2013b; Zakaria, Chin, & Daud, 2010), or TGT (Ismail, 2000; Ke & Grabowski, 2007), but also the appearance of specific methodologies for this scope, like “Small

Group Learning and Teaching in Mathematics” by Davidson (1990b), “Team Assisted Individualization” by Slavin (Slavin, Leavey, & Madden, 1984, 1986), “Learning Together” by Johnson and Johnson (Johnson & Johnson, 1991; Özsoy & Yildiz, 2004) or “Cooperative-Individualized Learning Approach in Mathematics” by Serrano (Serrano, González-Herrero, & Pons, 2008). This has triggered a line of research organized around the comparison of the effectiveness of different cooperative learning methods on specific intra-subject variability and, especially, on students' performance on mathematics (Awofala, Fatade, & Ola-Oluwa, 2012; Parchment, 2009; Syahrir, 2011). The mentioned positive effects of the cooperative organization of the math class are proven right, as much in real settings as in virtual ones, in such a way that a promising new line of research on synchronous and asynchronous learning which relies on Information Technologies as an essential backup for cooperation has appeared (Rubia & Guitert, 2014). In this regard, the interactive video, tablet computers, digital pencils, the interactive board or personal computers have turned out powerful tools for the implementation of cooperative learning approaches (Álvarez, Salavati, Nussbaum, & Milrad, 2013; Jackson, Brummel, Poliet, & Greer, 2013; Tsuei, 2012), giving rise to a new, specific line of work known as CSCL (Computer-Supported Cooperative Learning).

However, even though the positive effects of cooperation on academic performance are conclusively proven (Roseth, Johnson, & Johnson, 2008), the effectiveness of any cooperative methodology depends on an optimal harmonization between three structures (task, goal and reward) and two parameters (equality and mutuality). *Task Structure* is understood as the ways the different learning activities and assessment are organized and the ways students have to perform them so as to achieve the goals established and the competences set; *Reward Structure* makes reference to how the

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consequences resulting from the task performance level and finally, *Goal Structure* determines the purpose of teaching and learning (Johnson & Johnson, 1974; Vedder & Veendrick, 2003). As for the parameters, *mutuality* is understood as the

degree of connection and depth and directionality of the communicative transactions between students and *equality* as the degree of symmetry between the roles performed by the participants in a group learning activity.

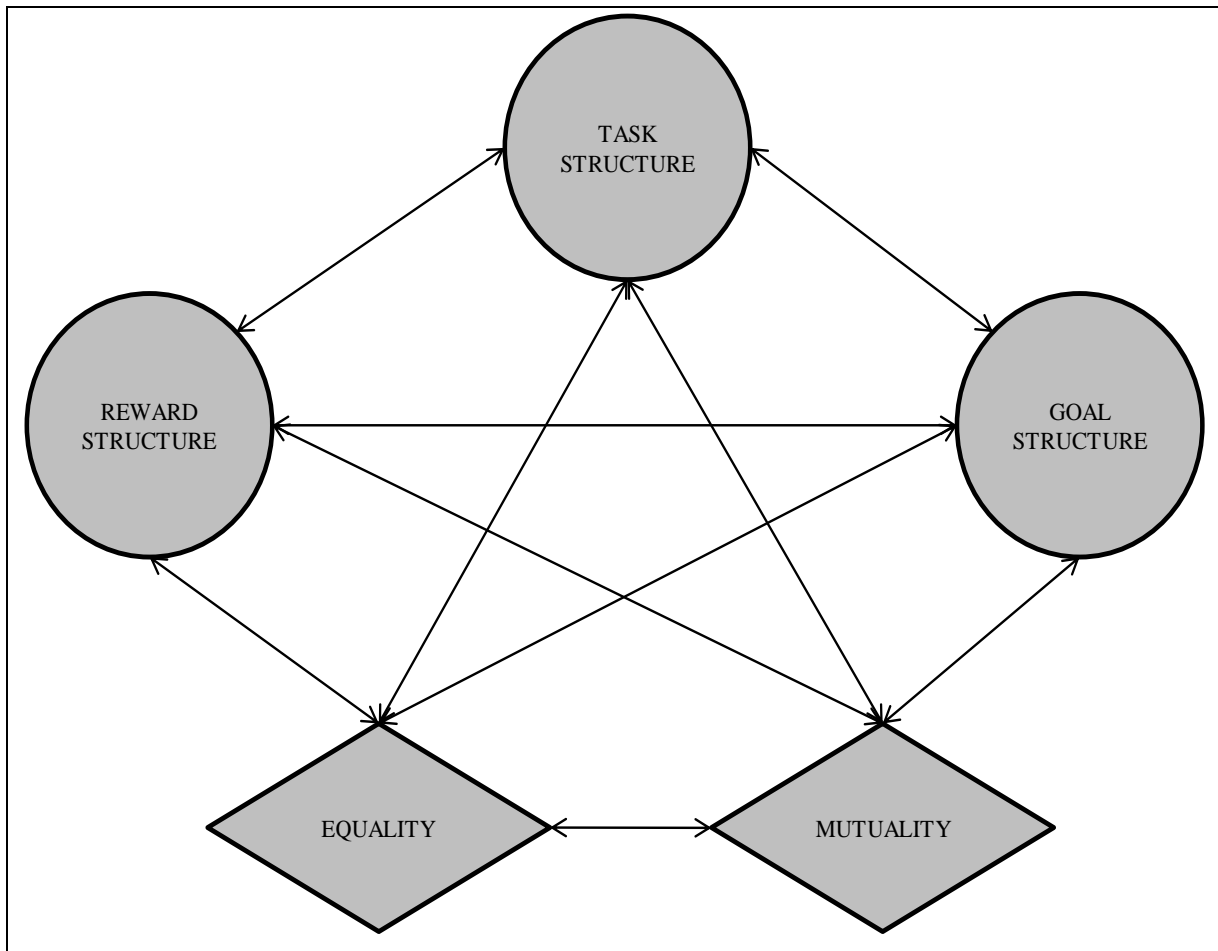


Figure 1. Structures and parameters that determine the effectiveness of a cooperative methodology.

Even though these five elements are highly interdependent, many a research tackle them independently so as to determine their effects on the academic performance in cooperative learning activities in the math class (Baliya, 2013; Hennessey & Dionig, 2013; Johnson & Johnson, 1974; Serrano & Pons, 2013; Shindler, 2010; Slavin, Hurley, & Chamberlain, 2003; Vedder & Veendrick, 2003), but very few provide inter-structural studies to compare the differentiated effects of each of the structures on academic performance, be it in interaction with the content, be it in interaction with the educational level or with any other variable which fosters an efficient use of cooperative learning (Gillies, 2003; Serrano & Pons, 2007). And if the inter-structural studies are scarce, the number of inter-parametrical investigations is even smaller and almost all of them revolve around the parameter of mutuality (Gagné & Parks, 2013; Kotsopoulos, 2014; Pons et al., 2012), being the comparative research between the three

relationships of cooperation which make up the parameter of equality practically nonexistent.

Indeed, regarding the parameter of equality, most authors, continuing the tradition started by Damon and Phelps (1989), distinguish between three types of relationships (O'Donnell & Hmelo-Silver, 2013): peer tutoring (low level of equality), collaboration (high level of equality) and cooperation (variable level of equality, without reaching extreme values).

Peer tutoring is a relationship centered on the transmission of information and established between students that, when presented a particular topic, show different levels of competence. It is based on a student/teacher pseudo-relationship which benefits from the socio-cognitive proximity between the elements of it and has an asymmetric interactive structure with a clear differentiation between the roles. For this relationship to succeed it is necessary that the task ensures

an appropriate level of participation and that the conditions below are met:

- The creation of a bridge of communication between students (via questions and answers).
- The organization of a structure for the resolution of problems inherent to the task.
- Transference of responsibility
- Co-participation in the resolution.
- Implicit or explicit interaction.

A substantial proportion of the essays on cooperative learning in mathematics have as a starting point this very type of relationship (Mesler, 2009; Topping & Bamford, 2012).

The *cooperation* is a relationship centered on the acquisition or application of a particular knowledge established between students who, when dealing with a specific subject, display different levels of competence while having certain degree of proximity. In this relationship, the roles need to be hierarchically equivalent and their differentiation should be made in connection with the task structure. The interactive structure is symmetric but, as the communication is aimed at different sources of information, it isn't devoid of certain asymmetry. The procedure control falls on the group, and the task, organized along the attainment of an only product, presents a structure susceptible of division (Hossaina & Tarmizi, 2013).

The *collaboration* is a relationship centered on the acquisition or application of a particular knowledge and it is established between students who, when presented a specific subject, show similar levels of competence. In this type of relationship, there is not a differentiation of roles and if there was one, these would need to be hierarchically equivalent. The procedure control falls on the group. The task, organized along the attainment of an only product, should present a unit structure (there is no task division), but it is susceptible of complementary interventions (Goos, 2004; Horn, 2012).

The decision making process on which of these relationships would be more worthwhile in the area of mathematics and their possible effectiveness in interaction with the type of mathematical content or the educational level has little empirical support (Eskay, Onu, Obiyo, & Obidoa, 2012; Taylor & MacKenney, 2008). Likewise, the role that the students' previous knowledge is an issue that has had little repercussion on the research on cooperative learning (Ertl & Mandl, 2006; Oortwijn, Boekaerts, Vedder, & Strijbos, 2008; Van Blankenstein, Dolmans, Van der Vleuten, & Schmidt, 2013), and even less to determine the most appropriate type of interactive structure for the initial level of competence at which our students start from when dealing with a particular task.

In this context, two questions on research, whose answer would be very relevant for the teaching praxis from the perspective of a cooperative organization of the classroom and which make up our work hypotheses arise. Firstly, if we take

into consideration the non-isomorphic nature of the structure of mathematical knowledge (algebraic knowledge, numerical knowledge, geometric knowledge, etc.), then, could we conclude that certain kinds of mathematical knowledge can be better organized and elaborated by the students using different interactive structures? And secondly, if that possibility of organizing and elaborating information depends upon the level of development and structuring of the student's mental schemes, could we, therefore, assume that the students' previous knowledge could be a confounding variable to estimate the most appropriate interactive structure?

Method

Design

In order to have our research enquiries answered and aim at verifying our experimental hypotheses, we suggest an inter-subject factorial design with three experimental groups and with pre and post-treatment measurements. The choice of this particular design derives from the fact that we intend to confirm the effects on academic performance that the use of the three interactive structures likely to be used in a cooperative organization of the classroom (treatment variable) carried out on equivalent groups has, trying to minimize the influence of the students' previous knowledge on the response variable.

Participants

The participants in the experiment were 72 students of the third year of Compulsory Secondary Education (ESO in Spanish) selected through intentional sampling, so everyone would receive the same type of treatment by the same teacher, thus eliminating a possible threat to the internal validity of the experiment. These students were affiliated with three groups of a state high school. The first of them was formed by 25 students, 13 boys and 12 girls with an average age of 15.03 years; the second group was made up by 24 students, 13 boys and 11 girls, with an average age of 15.02 years, and the third group was constituted by 23 students, 13 boys and 10 girls, with an average age of 15.02 years. Each of the groups was randomly assigned to each of the experimental conditions.

The students were informed that an empirical work to determine which methodology would be the most effective for teaching of mathematics was to be carried out and gave their total consent to take part in said experience.

Procedure

The experience took place during an academic year and the contents were structured along six blocks which matched, as much as possible, the contents covered during the previous academic year, thus, making it possible to determine the previous knowledge that the students had on

each one of the blocks to be developed. Since block n° 3 did not match the contents taught in the 2nd year of ESO (Compulsory Secondary Education), a specific test aimed at de-

termining their knowledge and administered at the beginning of the experiment was elaborated.

Table 1. Distribution of participants.

GROUP	RELATION	NUMBER (size)	TEAM	GENDER		AGES			
				M	F	Min	Max	Mean	SD
1	Collaboration	25	5(4) y 1(5)	13	12	14.5	15.5	15.03	TOTAL
2	Cooperation	24	6(4)	13	11	14.6	15.5	15.02	TOTAL
3	Peer-Tutoring	23	5(4) y 1(3)	13	10	14.7	15.5	15.02	TOTAL
TOTAL		72		39	33	14.5	15.5	15.02	0.250

Table 2. Prior Knowledge and Contents' distribution by Bloks.

BLOCKS	PRIOR KNOWLEDGE Secondary Education 2 nd	INSTRUCTIONAL CONTENTS Secondary Education 3 rd
Block 1 (numerical)	Divisibility, Whole Numbers, Fractions, Proportionality, Sexagesimal System	Fractions, and Rational Number
Block 2 (algebraic)	Polynomials, and Linear Equations (degree one)	Polynomials, Linear Equations (degree one), Systems or Lineal Equations, and Quadratic Equations (degree two)
Block 3 (sucessions)		Successions, and Progressions (arithmetic and geometric)
Block 4 (functions)	Graphics, and Functions	Graphics, and Functions: Increasing, Decreasing, Enclosed Functions, Symmetric Functions, Periodic Functions
Block 5 (geometry)	Triangles, Similarity, Euclidean Geometry, Figures Geometrics: Areas and Volumes	Euclidean Geometry (Translations, Rotations, and Symmetries). Figures Geometrics: Areas and Volumes
Block 6 (statistics)	Statistics	Probability and Statistics

First, and after assigning each one of the three experimental conditions to each of the groups, the experiment was started with two preparatory sessions for the correct development of the test, taking as a reference the identification performed by Hsiung, Luo, and Chung (2014) on effective and non effective cooperative teams and the ideas contributed by Lau (Lau, Kwong, Chong, & Wong, 2014), in order to determine the necessities skills for an optimal team work by means of cooperative methodology. In these sessions it was specified how the interaction should be produced and how the task had to be structured.

Each of the lessons proceeded as follows: first, the teacher went through the topic and exercises and activities where it was intended the participation and implication in situations of authentic learning were carried out (Herrington, Reeves, & Oliver, 2014). Next, students were given an envelope containing two questions and three problems to be solved and another envelope which contained the solutions highly detailed. Once the activity of the first envelop had been accomplished, they opened the envelope containing the solutions and self-corrected those exercises. Both the realization task as the self-correcting one were performed following the guidelines they had been given in the initial session. Finally, after developing every topic of a block of contents, an assessment of an individual nature containing three questions and five problems was undertaken.

The first group, affiliated with the experimental condition “*collaboration*”, worked using a non divisible task structure, so the different activities had to be performed collectively, checking the solutions and correcting them all together, asking the teacher for help in case any of the instructions was not understood or could not work out the activities by

themselves. The second group, affiliated with the experimental condition “*cooperation*”, worked by means of dyadic interaction, in such a way that while one pair worked out the questions, the other pair did so with the problems. Once the solutions had been elaborated, they were corrected (self-correcting envelope) and next, each one of the pairs explained to the other their work, detailing step by step how they had come up with the solution. If any doubt came up during the process, they would ask the teacher for help. For the next topic, they would change roles, so that the pair that had worked out the questions had to do likewise the problems and vice versa. Finally, in the third group, affiliated with the experimental condition “*peer-tutoring*”, the student-tutor had to work out the questions and problems, explaining in great detail how and why he did it in such way. Then, the tutored ones, in their groups, worked them out while the tutor supervised them and asked their peers the reasons that had led them to the solution of the problem. If the student-tutor had any doubt during the process, he/she would ask the teacher for help. Once the problem-solving process had finalized, they would move on to the self-correcting process.

In any of the three experimental conditions, when a given question was not understood during the self-correcting process, the teacher was asked to explain the steps which led to solving it.

Data Analysis

The data was subject to statistical treatment by means of a Unifactorial Covariance Analysis (ANCOVA) so as to eliminate the heterogeneity that could cause in the dependent variable (academic performance) the influence of stu-

dents' previous knowledge (mark obtained in the previous academic year), which worked as a covariate. This omnibus ANCOVA was broken down into six Covariance Analyses, one per each block of contents, in order to determine the effects of the treatment on each of those blocks. In these six ANCOVA's, the covariate (previous knowledge) was the marks obtained by the students in the previous academic year (PK2), divided into blocks of contents, with the exception of block n° 3, for which, as no equivalence with the contents of the 2nd year of ESO existed, an ad hoc initial knowledge assessment test was designed. In all the ANCOVA's the corresponding post-hoc analyses were conducted so as to perform the intergroup comparisons in twos.

Prior to the Covariance Analysis, the corresponding pre-test data were compared so as to determine if the groups, analyzed in twos, could be considered equivalent at the beginning of the experience (T-Test). The data was analyzed with SYSTAT 13.1 software package.

Results

The results obtained in the omnibus ANCOVA show significant differences, as much in the treatment variable RELATION ($F_{2, 68} = 12.103, p < .000; \eta_p^2 = .263$), as in the covariate PK2 ($F_{1, 68} = 122.272, p < .000; \eta_p^2 = .642$)¹.

These results prove the existence of different levels of academic performance between the three treatment groups.

To determine the meaning and magnitude of these differences, we relied on post-hoc comparisons between the three groups and found that the relationship of peer-tutoring (group 3) is considerably superior to the relationships of cooperation (group 2) ($p < .001$) and collaboration (group 1) ($p < .01$), being the difference between the two latter groups insignificant ($p_{1,2} = .581$).

However, when we performed the analysis sorted by blocks of contents, we came to very different interpretations.

Block 1: The participants in the experience did not show inter-group differences when at the beginning of it, that is, they were equivalent with regard to previous knowledge.

The results of the ANCOVA prove that the differences found in the academic performance can only be attributed to the students' previous knowledge ($F_{1, 68} = 4.558, p < .05$), as no significant differences in the treatment variable exist. The effect size is as well very modest ($\eta^2 = .040$).

The post-hoc comparisons confirm the absence of significant differences between the three groups ($p_{1,2} = .996, p_{1,3} = .289$ and $p_{2,3} = .339$).

Block 2: The inter-group comparisons with regard to the students' initial knowledge on this block showed no significant differences between the three treatment groups ($p_{1,2} = .993, p_{1,3} = .931$ and $p_{2,3} = .966$).

Just like in Block 1, the results of the ANCOVA show that the differences found in academic performance can only be attributed to the students' previous knowledge ($F_{1, 68} = 89.460, p < .001$), as the F value found for the treatment variable did not turn out significant with a size effect of $\eta^2 = .008$.

The post-hoc comparisons ($p_{1,2} = .994, p_{1,3} = .819$ and $p_{2,3} = .766$) prove this fact.

Block 3: Unlike the two former blocks, the contents of this one turn out quite new for the student. The test was designed so as to determine the students' previous knowledge, and though it reveals a high degree of homogeneity in the groups, as in no case did the differences turn out to be significant ($p_{1,2} = .594, p_{1,3} = .878$ and $p_{2,3} = .883$), some small differences in favor of group 1 (collaboration) when compared to groups 2 (cooperation) and 3 (peer-tutoring) did appear.

The Analysis of Covariance performed reveals that the differences between the groups is highly significant ($F_{2, 68} = 26.229, p < .001$), being the size effect ($\eta^2 = .425$). Likewise, the covariate (previous knowledge) shows a high explicative value ($F_{1, 68} = 33.933, p < .001$).

The post-hoc comparisons attest that these differences in the subjects' performance can be observed in all the possible comparisons, appearing a clear superiority of the relationship of tutoring over the relationship of cooperation and of the latter over the relationship of collaboration ($p_{1,2} = .008, p_{1,3} = .000$ and $p_{2,3} = .000$).

Block 4: The inter-group comparisons drawn before the implementation of the block do not reveal significant differences between the three treatment groups ($p_{1,2} = .813, p_{1,3} = .946$ and $p_{2,3} = .956$), so it could be concluded that the three groups started out in a situation of equality and that the differences likely to be found could be attributed to the treatment.

The ANCOVA for this block shows significant differences between the three types of inter-group relationships presented in our work ($F_{2, 68} = 9.869, p < .001$), with a size effect of .225.

However, a posteriori comparisons show that said differences do not exist between groups 2 and 3 ($p_{2,3} = .991$), but they do exist between these two groups and the group subject to treatment by means of collaborative work. Therefore, we could admit that with regard to these contents, the relationships of cooperation and tutoring are proven superior to the relationship of collaboration ($p_{1,2} = .001, p_{1,3} = .001$).

Block 5: The initial comparisons do not reveal significant differences between the three groups regarding initial knowledge, so we could acknowledge an initial inter-group homogeneity.

The analysis of covariance shows that, for this block of contents, the differences in treatment turn out to be very significant ($F_{2, 68} = 14,906, p < 0,001$), being the size effect 0.305.

In order to determine the meaning and intensity of these differences, the opportune post-hoc comparisons were per-

¹ The interpretation of the Effect Size from partial η^2 index follows the rule: “.02 as small, one of .13 as medium, and one of .26 as large” (Bakeman, 2005, p. 383).

formed and the results reveal significant values favoring the relationship of collaboration ($p_{1-2} = .05$ y $p_{1-3} < .001$). Likewise, the performance in the relationship of cooperation was superior to that in the relationship of peer-tutoring ($p_{2-3} = 0.05$).

Block 6: In the last of the block of contents, just like in the previous ones, no significant intergroup differences were found at the beginning of the activities ($p_{1-2} = .995$, $p_{1-3} = .890$, $p_{2-3} = .847$).

The ANCOVA corresponding to this block shows highly significant differences between groups ($F_{2, 68} = 28.016$, $p < 0.001$), with high values in size effect ($\eta^2 = 0.452$) and great significance for the covariate “previous knowledge” ($F_{1, 68} = 43.661$, $p < 0.001$).

The post-hoc comparisons show a clear disadvantage of the group which worked in the relationship of tutoring, compared to the one which worked in relationship of collaboration and, especially, in relationship of cooperation ($p_{1-2} = .056$, $p_{1-3} = .000$, $p_{2-3} = .000$).

Discussion and conclusions

The research which we have carried out derives from the widely proven premise that cooperative learning in any of its three interactive structures (collaboration, cooperation and tutoring) is superior to other classroom management strategies, and especially in the math class (Hossain & Tarmizi, 2013, Ke & Grabowski, 2007, Lehrer & Lesh, 2013, Nunnery, Chappell, & Arnold, 2013, Özsoy & Yildiz, 2004, Roseth, Johnson, & Johnson, 2008, Zakaria, Solfitri, Daud, & Abidin, 2013) for its evident influence on the generation mathematical reasoning (Booisen & Grosser, 2014). In fact, barring few studies which do not reveal differences between cooperation strategies and other types (Tracey, Madden, & Slavin, 2010), most works confirm that the cooperative organization in the math class it is indeed the teaching and learning structure which generates the best results in a highly relevant variable for this particular discipline: academic performance (Winne & Nesbit, 2010).

Even though the post-hoc comparisons performed on the results obtained from the F-omnibus of the global ANCOVA could match with those obtained in other studies where peer tutoring emerged as a powerful tool (Eskay, Onu, Obiyo, & Obidoa, 2012, Mesler, 2009, Topping & Bamford, 2012, Tsuei, 2012), the six intra-content analyses allow us to elaborate very relevant conclusions for the implementation of a cooperative methodology in the math class.

In effect, the structure of taught skills is organized along six blocks with specific and differentiated characteristics when related to the contents learned in the previous academic year. Firstly, we find two blocks which repeat contents from the previous year: block 4 (Graphics and Functions) and block 5 (Geometric Figures: areas and volumes). Secondly, we find two other blocks that, while not repeating contents developed throughout the former year, are consti-

tuted by topics whose bases were laid down during that year: block 1 (Rational Numbers), as the notions of *Whole Number* and *Fraction* were object of study in the previous year, and block 2 (linear equation systems and quadratic equations), as first degree equations were also studied during the previous academic year. Finally, we two blocks whose contents are completely new for students: block 3 (summations and progressions) and block 6 (probability).

Firstly, when the contents are not new for students (Blocks 4 and 5) the most effective intra-group relationship is the “relationship of collaboration” and the least effective, the “relationship of tutoring”. The explanation to this phenomenon can be found in the fact that students taking part in that relationship had a significant knowledge beforehand as they all had proven sufficient command on the knowledge of said block contents in the previous academic year. In this case, the instructional context demands a type of learning where the cognitive needs only demand a development of the “growth” and “adjustment” schemes, as the modifications to be produced in the student’s cognitive structure do not imply a structural change, as the concepts and procedures to be initiated by the student so as to solve the situation/problems are essentially the same, albeit with a higher degree of complexity and precision and, at most, only demands the differentiation of schemes or the integration of these schemes in other previously existing, but more general ones. Under these conditions the conceptual controversies arising during the process of interactivity which result from a situation of collaboration can be easily overcome, getting to situations of coordinate solution which, given the interactive nature of the dynamics of said situation, getting to a higher level of finesse, going further in their knowledge than when dealing a situation where the “tutor” guides their performance, and thus, their individual contributions towards the search of a solution are substantially lesser. This way, we could conclude that, when the learning situation does not demand a conceptual change the best interactive structure so as to organize the math class is the relationship of collaboration.

Secondly, when the student deals with contents that, while being new, have had their bases laid down during the previous year, as in the case of blocks 1 and 2, we found that no significant differences in performance appeared between the three interactive structures. In this case the explanation lies in the fact that students have a degree of previous knowledge which enables them to deal with the new contents with guarantee of success, as long as the mechanisms which make possible the interactivity lead to processes of knowledge hierarchy reordering. A reorganization of this type, even when it does not force students to a theoretical change, does imply a qualitative transformation of their implicit theories. In effect, the new contents: Rational Numbers (block 1), linear equation systems/second degree equations (block 2) entail, respectively, a hierarchical reordering performed derived from the students’ previous knowledge (Fractions and Proportionality in the case of block 1, Linear

Equations in block 2), but the theories which underlie their knowledge hold up. For instance, the “theory on equation solving” which students apply on activities with new contents remains intact, because it is useful to them, so as to work out different types of equations. In these situations, while it is true that students have the necessary cognitive instruments so as to perform the structural reorganizations which the new contents demand and that these reorganizations could be performed in the course of interactivity processes emerging in interactions with classmates of a similar level of previous knowledge structuring, it is also equally true that interactions revolving around the new content which included more skilled students, could act as efficient aid mechanisms to accomplish the cognitive restructuring that the new contents require. Therefore, we could conclude that when the learning situation demands an insignificant conceptual change, the three interactive situations (*collaboration, cooperation and tutoring*) turn out to be equally effective.

Thirdly, when students have to deal with contents where their previous knowledge proves to be insufficient to respond to the demands that the new learning situations require, as in the case of block 3 (Arithmetic and Geometric Progressions) and block 6 (Probability), then the relationship of tutoring is revealed as the most efficient interactive structure. In effect, in these two blocks the acquisition of new knowledge implies a process in which the relationships between schemes would have to change drastically, in such a way that the schemes themselves would acquire new meanings, which in turn involves a theoretical change that would

alter the student’s explanations on real life phenomena, in Norman’s words (1982) we would be facing a “strong conceptual change”. In this case, the aid petitions play a fundamental role for the student’s cognitive progress and for this reason the relationship of tutoring is proven superior to the rest of the situations of interaction. This way we can conclude that when the learning situation forces the student to implement a strong conceptual change, the intra-group asymmetry, based on effective aid, fosters the appearance of effective learning.

For all this, it would not be daring to affirm that students’ previous knowledge are not only relevant as a starting point for the process of acquisition of new knowledge by means of cooperative work because in situations of learning between equals, a more positive elaboration of the new contents is produced when students start off with a high previous knowledge (Van Blankenstein, et al., 2013), but it also turns out to be decisive when it comes to determining the most appropriate interactive structure to set a cooperative organization in the math class. These results contradict, partially, the ones found by other researchers (Ertl & Mandl, 2006), which establish a disjunction between cooperative learning and previous knowledge

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