

Independence of basic arithmetic operations: Evidence from cognitive neuropsychology

María P. Salguero-Alcañiz and Jose R. Alameda-Bailén*

Área de Psicología Básica de la Universidad de Huelva

Título: Independencia entre operaciones aritméticas básicas: evidencia desde la neuropsicología cognitiva.

Resumen: Los casos descritos en la literatura ponen de manifiesto que las operaciones aritméticas pueden funcionar independientemente, lo que permite inferir que los procesos cognitivos implicados en las distintas operaciones podrían ser distintos.

El objetivo de este trabajo es determinar los distintos procesos implicados en la resolución de operaciones aritméticas: suma, resta y multiplicación.

Método. Instrumento: Batería de evaluación del procesamiento numérico y el cálculo (Salguero y Alameda, 2007, 2011). Sujetos: pacientes con daño cerebral adquirido.

Resultados y conclusiones: El paciente MNL conserva la suma y la multiplicación pero presenta alterada la resta. Por el contrario, el paciente PP manifiesta alteraciones en la suma y multiplicación pero conserva intacta la resta. ISR presenta un déficit selectivo para la multiplicación estando intactas la suma y la resta. Por último, ACH, conserva la suma pero tiene alteradas la suma y la multiplicación.

Esta doble disociación confirma los postulados del modelo anatómico funcional de Dehaene y Cohen (1995, 1997), que plantea la existencia de una doble vía para la resolución de operaciones aritméticas simples: la ruta lingüística, para datos numéricos aprendidos memorísticamente, que se utilizaría para sumar y multiplicar, y por otro lado, la elaboración semántica, para la resta.

Palabras clave: Daño cerebral; cálculo; neuropsicología cognitiva; doble disociación; operaciones aritméticas.

Abstract: The cases described in literature evidence that arithmetical operations can function independently, which allows to infer that the cognitive processes involved in the different operations might be different.

Objective of that work is to determine the different processes involved in the resolution of arithmetical operations: addition, subtraction and multiplication.

Method. Instrument: Assessment of Numeric Processing and Calculation Battery (Salguero & Alameda, 2007, 2011). Subjects. Patients of acquired cerebral injury.

Results and conclusions. The patient MNL preserves the addition and the multiplication but he presents altered the subtraction. On the contrary, the patient PP shows alterations in addition and multiplication but he conserves the skills for the subtraction. ISR presents a selective deficit for multiplication with intact addition and subtraction. Finally, ACH preserves the addition but presents deficit for subtraction and multiplication.

This double dissociation confirms the postulates of the anatomical functional model of Dehaene and Cohen (1995, 1997) that consider a double route for the resolution of arithmetical simple operations: linguistic route, for numerical information learned automatically (of memory) and would be used for the operations of addition and multiplication, on the other hand the semantic elaboration would be for subtraction.

Key words: Brain injury; calculation; neuropsychology; double dissociation; arithmetical operations.

Introduction

There is some consensus that the cognitive mechanisms involved in numerical recoding tasks (reading Arabic and Verbal numerals, writing dictated Arabic and Verbal numerals, and recoding from Verbal to Arabic and from Arabic to Verbal code) are the same as those underlying linguistic processing in general and will therefore depend on the same cortical areas (Alsina & Sáiz, 2003; Dehaene & Cohen, 1995, 1997; Salguero, 2007). In other words, these linguistic functions of numbers are a part of language processing in general. The so-called lexical numerical knowledge is also related to these linguistic functions of numbers (Alameda, Salguero, & Lorca, 2007; Salguero & Alameda, 2003; Salguero, Lorca, & Alameda, 2004).

However, the processes involved in calculation are not as clear. This could be because diverse arithmetic operations depend on different cognitive processes. The antecedents described in the literature seem to point in this direction. Among them are patients RG (Dagenbach & McCloskey, 1992) and HAR (McNeil & Warrington, 1994), who preserved subtraction despite deterioration of addition and multiplication, and patients MAR and BOO (Dehaene & Cohen,

1997), with the former presenting alterations only in subtraction, and the latter in multiplication. Lastly, Van Harskamp, and Cipolotti (2001) describe the cases of three patients with exclusive alterations in one arithmetic operation: FS, with selective impairment for simple addition, VP with a specific alteration for simple multiplications, and DT with an exclusive deficit for simple subtractions.

All this empirical evidence seems to indicate that diverse arithmetic operations depend on different cognitive processes that are independently susceptible to injury and therefore, that basic arithmetic operations function independently. However, the explanation of this fact varies depending on the different theoretical models.

The Triple Code model (Figure 1) proposes that numbers can be mentally represented in three different types of codes:

Visual-Arabic number form. This is the representation of the number in Arabic form and is therefore visual in nature.

Auditive-Verbal word structure, created and manipulated by general the language modules, it is the sequence of words associated with the number.

Analogical magnitude representation, in which numeric quantities are represented as distributions of activation on an analogical number line oriented left-to-right (or vice versa, depending on the culture) that fulfills Weber's psychophysical law.

*** Dirección para correspondencia [Correspondence address]:**

Jose Ramón Alameda Bailén. Área de Psicología Básica de la Universidad de Huelva. Avda. de las Fuerzas Armadas, S/N (Campus de El Carmen). 21071 Huelva (España). Email: alameda@uhu.es

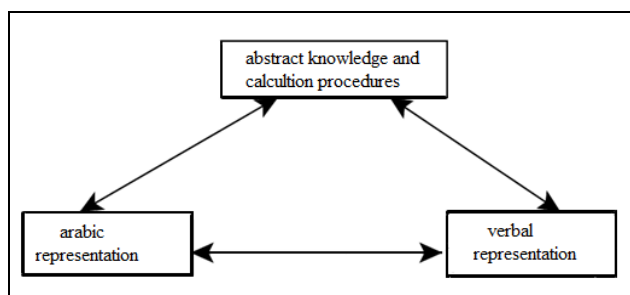


Figure 1. Triple Code Model of Numerical Cognition (Dehaene, 1992).

Each numeric procedure is linked to a specific input and output code. That is, each numerical task can be decomposed into a sequence of processes requiring a specific numerical input format. The format in which numbers are manipulated can be independently assessed for each task component.

The use of each type of code depends on task demands. Thus, the auditive-verbal code allows encoding numerals in verbal form, and is used, for example, to count. The visual-arabic code encodes numerals in Arabic notation and is mainly used for calculation operations with several written digits. Lastly, the analogical magnitude representation is used to manipulate quantities, for example, in a numerical comparison task, or to perform estimates.

The anatomical-functional model (Dehaene & Cohen, 1995, 1997), based on the Triple Code model, also accepts the existence of the same types of mental representations for numbers. The novelty of this model (Figure 2) is that it locates each type of representation in a certain brain area, so the functional postulates are the same as those of the Triple Code, that is, it postulates the existence of three types of representations that allow the manipulation of numeric symbols.

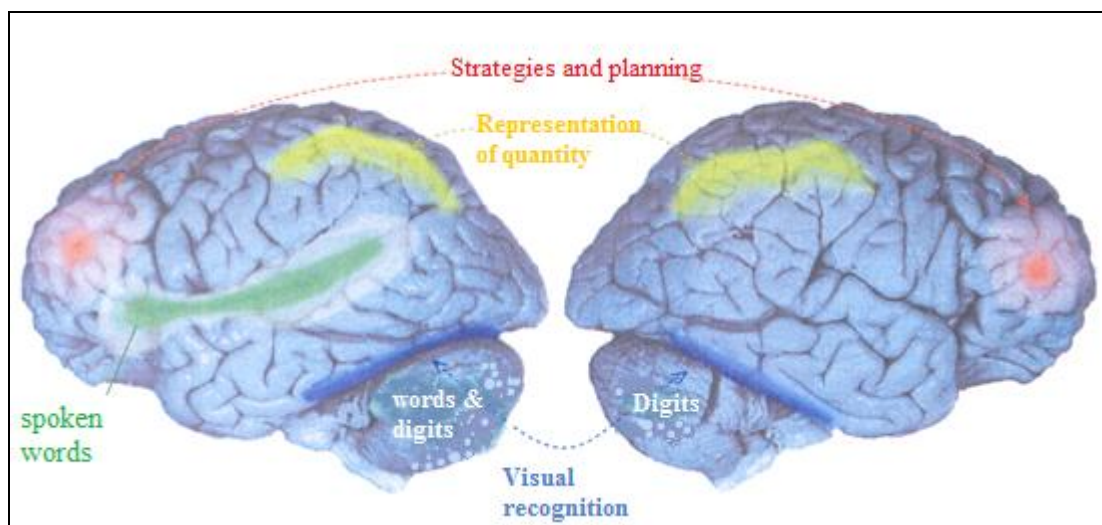


Figure 2. Anatomical Functional Model of Number Processing (Dehaene & Cohen, 1995, 1997).

Therefore, the Anatomical-Functional model (Dehaene & Cohen, 1995) implies the anatomical application of the Triple Code model (Dehaene, 1992). The main postulates are as follows:

1. Both hemispheres have visual identification mechanisms. The visual system of the left hemisphere can recognize all the simple digits, numerals of several digits, and written words. The end result is a representation of the identities and the relative position of the symbols or groups of symbols of the stimulus. This has been called the “visual form of the number” for Arabic numerals (Cohen & Dehaene, 1991). Anatomically speaking, in the left hemisphere, this system is located in areas of the occipital-temporal region, belonging to the “ventral visual stream,” which is responsible for visual recognition (Ungerleider & Mishkin, 1982). The counterparts in the right hemisphere can also identify visual symbols such as Arabic digits, numerals with various digits, and some words.

2. Both hemispheres have an analogical representation of quantities or numerical magnitudes. These processes are located in cortical areas of the parietal-occipital-temporal intersection in both hemispheres, although the right hemisphere can process quantities better than the left one (Kosslyn et al., 1989).

3. Only the left hemisphere can represent the sequence of words corresponding to verbal numerals and the procedures in order to identify and produce numerals orally. These procedures, which are not specific to numbers and which are situated in the classic language areas of the left hemisphere, therefore include the inferior frontal and superior and middle temporal gyri, as well as the basal ganglia and the nucleus of the thalamus.

4. Mental arithmetic is closely linked to language and to the verbal representations of numbers; that is, retrieval of arithmetical data from the memory is located in the language areas of the left hemisphere and this cannot be done by the

right hemisphere. Calculation procedures with numbers of various digits are more complex and imply the coordination of the visuo-spatial and verbal representations of digits.

5. In the left hemisphere, the visual, verbal, and magnitude representations are interconnected and can exchange information directly through recoding pathways. Specifically, the verbal system is directly connected to the visual identification system, so, a number can be named without the information having to be represented as magnitude; in this case, the recoding pathway is asemantic, whereas if the information must be represented as a magnitude, the processing pathway is semantic. Thus, the left hemisphere can recode numerals by means of two different types of processes, either through the asemantic pathway or through the semantic pathway. In the right hemisphere, visual and analogical magnitude representations are also connected with each other.

6. In healthy subjects, visual representations from both hemispheres are interconnected through the corpus callosum. Magnitude representations of both hemispheres are also interconnected through the corpus callosum. There are no other pathways to exchange numerical information between the two hemispheres. Therefore, there is no direct pathway between the visual form of the number in the right hemisphere and the verbal system, located in the left hemisphere. Visual-form information in the right hemisphere must pass through the corpus callosum to reach the visual form in the left hemisphere, and from there, access the linguistic system, which is exclusive to the left hemisphere.

In contrast, concerning numerical processing, the model of McCloskey (1992) proposes obligatory access to the magnitude represented by the number; that is, all the processes for switching from one code to another must be represented in the form of internal semantics. The model (Figure 3) proposes number processing as a system made up of different modules that operate autonomously, and each one of them is specialized in a certain function.

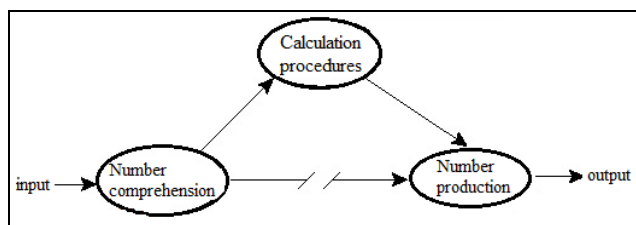


Figure 3. Schematic representation of Number Processing and Calculation Systems (McCloskey, 1992).

The mechanisms in charge of numerical production and comprehension operate at the same time, and each one of them is made up of syntactic and lexical processing units. Thus, there is a module for comprehension of Arabic numbers and another module for verbal numbers, and each one of them is made up of a lexical and a syntactic sub-process. Likewise, there is a module for the production of Arabic numbers and another one for Verbal numbers. This organi-

zation in independent modules explains why it is relatively easy to switch from one code to another: from comprehension of an Arabic number to production in Verbal form (oral or written), and vice versa.

In addition, this model postulates the existence of an internal representation between the processes of comprehension and production. Thus, independently of the code used, inputs and outputs must be represented, specifying the value of the number in abstract form. For example, number 72 indicates the quantity 7 times 10 plus 2 units. According to the model, in order to transform a number from a code, its value must be represented. This magnitude representation is abstract and common for all the modalities, both of inputs and outputs.

Hence, this model distinguishes three types of processes: comprehension mechanisms, production mechanisms, and internal semantic representations.

1. The mechanisms of numerical comprehension transform numerical inputs into abstract internal representations, which can be used for the next cognitive processes, such as calculation.

2. The mechanisms of numerical production transform the internal representations of numbers into the appropriate output format.

3. The semantic representations specify the basic quantities and associated powers of 10. For example, the semantic representation $\{5\}10^3$, $\{3\}10^1$ corresponds to the number 5,030. The digit in brackets indicates the quantity and then, the corresponding power of 10 is specified (10^n), for example, $\{5\} 10^3$, would be 5 times 10 to the third power, that is, 5,000.

With regard to the system of simple calculation, McCloskey, Caramazza, and Basili, (1985) propose that, in addition to the described numerical processing mechanisms, any calculation task requires the following specific cognitive mechanisms:

1. Arithmetic sign processing, either written symbols (for example: +, -, *, ÷) or words (for example, plus, minus).

2. Retrieval of basic arithmetic data (for example, the data from tables such as $6*7=42$).

3. Execution of the calculation procedure. For example, when adding various digits, the procedure consists of starting at the right column, retrieving the basic arithmetic datum from the sum of the digits of this column, writing the unit of the result of this first sum under the column, remembering that one has to "carry" if the result was higher than nine, and continue with the next column to the left, to which the "carried" number must be added, and so on.

In summary, according to this model, the calculation system is made up of three specific and autonomous elements: processing arithmetic signs, knowledge and retrieval of arithmetic data, and the calculation procedures. These three elements are susceptible to independent injury (e.g., Ferro & Botelho, 1980; Salguero & Alameda, 2010; Salguero, Lorca, & Alameda, 2003).

Method

Patients

A total of 45 patients with acquired brain damage were assessed, 19 of whom had some kind of alteration in the system of numeric processing and calculation. In this work, we shall focus on the results of four patients.

- MNL (male, 47 years) suffered infarct of the middle cerebral artery in the left hemisphere and presents executive dysfunction.
- PP (male, 54 years) has a temporal-parietal-occipital lesion in the left hemisphere and presents linguistic alterations.
- ISR (male, 16 years) has a diffuse axonal injury and presents mild memory deficit.
- ACH (male, 40 years) suffers diffuse axonal injury and presents attentional deficits and mild mnestic alterations.

Instrument

To examine the numeric and calculation skills, we employed the "Batería de Evaluación del Procesamiento Numérico y el Cálculo" (in English, the Assessment of Numeric Processing and Calculation Battery) of Salguero and Alameda (2007, 2011). This instrument is made up of six blocks that serve to assess the six important types of skills involved:

Block I: numerical comprehension

Block II: numerical recoding

Block III: arithmetic signs

Block IV: calculation

Block V: numerical lexical knowledge

Block VI: numerical sequence

Block IV, which assesses calculation, and which is dealt with in this work, is made up of the following tasks:

- Verification of results
- Numerical reasoning
- Written calculation: additions, subtractions, and multiplications of 1 to 3 digits, either carrying or not carrying.
- Oral calculation: additions, subtractions, and multiplications of 1 to 2 digits, either carrying or not carrying.

Statistical analysis

The data were statistically analyzed by means of a difference of proportions (Moore & McCabe, 2001; Pryce, 2005), comparing the patients' scores with those of a reference control group (Salguero & Alameda, 2007, 2011), obtaining a z-score and its significance. This procedure allows us to compare two scores, either with the same test or with parallel tests, or to compare a patient's performance with that of a control group, or to compare the performances of two different patients, even in situations where the total number of items used in each case is different.

Procedure

The patients were assessed individually in one-hour sessions. Pencil-and-paper tests were used.

Results

Tables 1 and 2 present the results (percentage of correct responses and significance) of these patients in all the battery tests.

Table 1. Results obtained in the first three blocks.

| Block I. Numerical comprehension | Patients | | | | | | | |
|--|----------|----------|------|----------|------|----------|------|----------|
| | MNL | | PP | | ISR | | ACH | |
| | % CR | <i>p</i> | % CR | <i>p</i> | % CR | <i>p</i> | % CR | <i>p</i> |
| 1. Identification of Arabic numbers | 100 | | 95 | .23 | 100 | | 100 | |
| 2. Numerical comparison | 94 | .01 | 100 | | 100 | | 100 | |
| 3. Numerical bisection task | 71.5 | .02 | 100 | | 100 | | 78.5 | .05 |
| 4. Numerical proximity task | 100 | | 100 | | 100 | | 100 | |
| 5. Simple number-quantity verification | 85.5 | .21 | 100 | | 100 | | 100 | |
| 6. Number-quantity association with written production | 85.5 | .21 | 100 | | 100 | | 100 | |
| 7. Association verification | 100 | | 100 | | 92 | .23 | 100 | |
| 8. Analogical scale: thermometer task | 100 | | 100 | | 100 | | 100 | |
| Block II. Numerical Recoding | | | | | | | | |
| 9. Repetition of names of numbers | 91 | .01 | 88.5 | .00 | 100 | | 100 | |
| 10. Reading Arabic numbers | 97 | .15 | NR | | 100 | | 100 | |
| 11. Reading Verbal numbers | 100 | | NR | | 100 | | 87.5 | .01 |
| 12. Arabic-Verbal recoding | 80 | .00 | 100 | | 100 | | 100 | |
| 13. Verbal-Arabic recoding | 100 | | 96 | .45 | 100 | | 78 | .00 |
| 14. Writing dictated numbers (in Verbal form) | 100 | | NR | | 100 | | 100 | |
| 15. Writing dictated numbers (in Arabic form) | 100 | | 65 | .00 | 100 | | 87.5 | .1 |
| Block III. Arithmetic signs | | | | | | | | |
| 16a. Arithmetic signs: identification | 100 | | 100 | | 100 | | 100 | |
| 16b. Arithmetic signs: naming | 100 | | 0 | .00 | 100 | | 100 | |
| 17. Use of arithmetic signs | 68.5 | .01 | 62 | .00 | 100 | | 100 | |

Note: CR = correct responses. NR = no reply.

Table 2. Results obtained in Blocks IV to VI.

| | Patients | | | | | | | |
|---|----------|----------|------|----------|------|----------|------|----------|
| | MNL | | PP | | ISR | | ACH | |
| | % CR | <i>p</i> | % CR | <i>p</i> | % CR | <i>p</i> | % CR | <i>p</i> |
| 18. Verification of results | 89 | .23 | 78 | .07 | 100 | | 78 | .07 |
| 19. Numerical reasoning | 50 | .00 | 100 | | 100 | | 63.5 | .00 |
| 20. Written addition | 94.5 | .55 | 69 | .00 | 97.5 | .99 | 94.5 | .55 |
| 21. Written subtraction | 73.5 | .01 | 96 | .66 | 97 | .66 | 66.5 | .00 |
| 22. Written multiplication | 93 | .86 | 66 | .02 | 69 | .00 | 53 | .00 |
| 23. Oral addition | 60 | .03 | 60 | .03 | 100 | | 66.5 | .06 |
| 24. Oral subtraction | 60 | .03 | 46.5 | .00 | 93.5 | 1 | 73.5 | .14 |
| 25. Oral multiplication | 50 | .05 | 3 | .00 | 70 | .26 | 40 | .00 |
| Block V. Numerical lexical knowledge | | | | | | | | |
| 26. Numerical lexical knowledge: questions | 83 | .87 | 58.5 | .01 | 81.5 | .76 | 73 | .23 |
| 27. Parity judgments | 100 | | 100 | | 100 | | 100 | |
| Block IV. Numerical sequence | | | | | | | | |
| 28. Reciting numerical sequence | 100 | | 100 | | 100 | | 100 | |
| 29. Numerical sequence: order | 100 | | 100 | | 100 | | 100 | |
| 30. Series of even numbers | 100 | | 100 | | 100 | | 100 | |

Note: CR = correct responses.

Firstly, we note that the diverse tasks assessed in the Calculation block seem to be independent. For example, patients MNL and ACH carried out the task of verification of results correctly but they presented alterations in the numerical reasoning tasks.

With regard to the arithmetic operations assessed by means of written tasks, the results of each patient were observed to vary as a function of the operation. Firstly, patient MNL retained the skills involved in solving addition and multiplication, but he presented impairment for subtractions ($\bar{\chi} = 2.3$, $p = .01$). In contrast, patient PP presented an inverse pattern, that is, he retained the skills for subtraction but he presented impairment for addition and multiplication ($\bar{\chi} = 3.16$, $p = .00$ and $\bar{\chi} = 2.18$, $p = .02$, respectively). But in patient ISR, we observed an impairment that exclusively affected multiplication operations ($\bar{\chi} = 2.18$, $p = .02$), while retaining addition and subtraction. Lastly, patient ACH was observed to retain addition, but the processes of subtraction and multiplication ($\bar{\chi} = 2.91$, $p = .00$ and $\bar{\chi} = 3.08$, $p = .00$, respectively) were impaired.

Discussion

Our results are in coherence with the studies described in the literature, in the sense that one arithmetic operation can be impaired after brain damage while others are retained. Hence, as arithmetic operations are susceptible to independent damage, it could be stated that they also function independently in healthy subjects.

Firstly, the pattern observed in our patient PP is also consistent with the results of previous works. PP shows evidence of impairment in addition and multiplication, but he retains subtraction. This same result was described by Dagenbach and McCloskey (1992) in patient RG, as well as by McNeil and Warrington (1994) in the case of patient HAR. As in these cases, in our patient ACH, more than one arith-

metic operation is affected, specifically subtraction and multiplication, but he retains addition.

However, our patients MNL and ISR present selective deficits only for one arithmetic operation. With regard to MNL, the impairment exclusively affects subtraction, while retaining addition and multiplication. This same performance pattern has been described in prior works, such as the case of MAR (Dehaene & Cohen, 1997) as well as the case of DT (Van Harskamp & Cipolotti, 2001). Cases of selective deficit for addition, such as that of patient FS of Van Harskamp and Cipolotti (2001), have also been described. Lastly, the cases like that of our patient ISR, with exclusive impairment for multiplication, coincide with those described by Dehaene and Cohen (1997) as well as that of VP (Van Harskamp & Cipolotti, 2001).

In summary, as in prior works, our results reveal that the diverse arithmetic operations can be selectively impaired as a consequence of brain damage, which means that in healthy subjects, these operations are independent of each other.

According to the theoretical models that attempt to explain the retrieval of numerical data when performing simple arithmetic operations, there are at least two positions.

On the one hand, according to the Anatomical functional model (Dehaene & Cohen, 1997), there are two different pathways to solve simple arithmetic operations, which will be used depending on the operation in question. There is the *direct pathway*, which is of an asemantic nature and therefore without access to the quantity represented by the number. According to the Triple Code model, this pathway depends on representations of auditive verbal numbers; that is, on the verbal representations of numbers and therefore, it allows solving arithmetic tasks that were previously learnt as verbal routines, which are mainly the addition and the multiplication. Therefore, it is a linguistic and rote type of processing. Operations of subtraction could not be solved through this pathway because they require access to the quantity represented by numbers as well as to processes of semantic

elaboration. Therefore, subtractions and divisions are carried out through an *indirect pathway*, which transforms the input (either visual Arabic or auditive verbal) into the corresponding analogical magnitude representation. That is, this indirect pathway is semantic and provides access to the quantity represented by the number; it thereby allows solving subtraction and division operations through semantic elaboration, as well as retrieval of any other numeric datum that was not rote learned.

With regard to our patient MNL, this model could explain his case, but only partially. It could be argued that the patient retains addition and multiplication because he preserves the auditive verbal representation of numbers, which would allow him to resolve these operations through the direct pathway. However, the deficit in subtraction is more difficult for the model to explain because, although it accepts that the analogical magnitude representation in the left hemisphere may be damaged, according to the data of the injury and following model, the analogical representation of numbers in the right hemisphere would remain intact.

With regard to PP, this model explains his performance pattern as a consequence of the alteration of the auditive verbal representation of numbers, located exclusively in the left hemisphere, which would prevent him from solving addition and multiplication problems, but he would retain the analogical magnitude representation in both hemispheres, so he could perform subtractions. This would be a clear example of impairment of the direct pathway and conservation of the indirect pathway.

With regard to patients ACH and ISR, explanations from the anatomical functional model are limited. ACH presents impairment of the processes related to addition, but he retains those of multiplication; this is difficult to explain from this model because it proposes that both operations depend on the same numerical representation, the auditive verbal one, and on the same processes to retrieve numerical data, that is, the direct asemantic and mnemonic pathway. The same obstacles are noted in the explanation of the case of ISR because he retains addition but presents impairment for multiplication.

References

- Alameda, J. R., Salguero, M. P., & Lorca, J. A. (2007). Conocimiento numérico cuantitativo y léxico: evidencia de doble disociación. *Psicothema, 19* (3), 381-387.
- Alsina, A., & Sáiz, D. (2003). Un análisis comparativo del papel del bucle fonológico versus la agenda visoespacial en el cálculo en niños de 7-8 años. *Psicothema, 15*(2), 241-246.
- Cohen, L., & Dehaene, S. (1991). Neglect dyslexia for numbers? A case report. *Cognitive Neuropsychology, 8*, 39-58. doi: 10.1080/02643299108253366
- Dagenbach, D., & McCloskey, M. (1992). The organisation of arithmetic facts in memory: Evidence from a brain-damaged patient. *Brain and Cognition, 20*, 345-366. doi: 10.1016/0278-2626(92)90026-1
- Dehaene, S. (1992). Varieties of numerical abilities. *Cognition, 44*, 1-42. doi: 10.1016/0010-0277(92)90049-N
- Dehaene, S., & Cohen, L. (1995). Towards an anatomical and functional model of number processing. *Mathematical Cognition, 1*, 83-120.
- Dehaene, S., & Cohen, L. (1997). Cerebral pathways for calculation: Double dissociation between rote verbal and quantitative knowledge of arithmetic. *Cortex, 33*, 219-250. doi: 10.1016/S0010-9452(08)70002-9
- Ferro, J. M., & Botelho, M. A. S. (1980). Alexia for arithmetical sign. A cause of disturbed calculation. *Cortex, 16*, 175-180.
- Kosslyn, S. M., Koenig, O., Barrett, A., Cave, C. B., Tang, J., & Gabrieli, J. D. E. (1989). Evidence for two types of spatial representations: Hemispheric specialisations for categorical and coordinate relations. *Journal of Experimental Psychology: Human Perception And Performance, 15*, 723-735. doi: 10.1037/0096-1523.15.4.723
- However, the model of McCloskey (McCloskey, 1992; McCloskey et al., 1985) considers that each arithmetic operation has its own representation and its own storehouse of numerical data, so it can explain the performance patterns observed in our patients. In the case of PP, both the processes related to addition and to multiplication are affected. Likewise, ACH would also have two different alterations, one affecting subtraction and the other, multiplication. Lastly, the cases of MNL and IRS, with exclusive impairments for one operation would be due to the impairment of that operation; that is, MNL would have impairment for the processes involved in subtraction whereas ISR would have impairment for multiplication.
- In summary, McCloskey's model proposes the existence of independent representations and processes for each arithmetic operation, which would clearly explain how they can be impaired selectively after brain damage.

Conclusions

First, as commented, the current empirical evidence is difficult to explain with the anatomical functional model (Dehaene & Cohen, 1997). This model presents two large limitations to address the performance patterns observed in patients. On the one hand, there is some rigidity in these impairment patterns that is not supported by the data. The model proposes that addition and multiplication depend on the same type of numerical representation, auditive and verbal, and on the same data retrieval mechanisms, that is through the direct linguistic and asemantic pathway. According to this, addition and multiplication should have the same status, either or both retained or both impaired. As seen, the results of this and other studies do not confirm this assumption.

Another limitation of the model is that it does not explain why the indirect semantic pathway cannot substitute the direct pathway, for example, in selective impairment of addition or of multiplication.

Therefore, the postulates of McCloskey (1992) fit the empirical data better, although it entails accepting different memory storehouses for the diverse arithmetic operations.

- McCloskey, M. (1992). Cognitive mechanisms in numerical processing: Evidence from acquired dyscalculia. *Cognition*, *44*, 107-157. doi: 10.1016/0010-0277(92)90052-J
- McCloskey, M., Caramazza, A., & Basili, A. (1985). Cognitive mechanisms in number processing and calculation: Evidence from dyscalculia. *Brain and Cognition*, *4*, 171-196. doi: 10.1016/0278-2626(85)90069-7
- McNeil, J. E., & Warrington, E. K. (1994). A dissociation between addition and subtraction within written calculation. *Neuropsychologia*, *32*, 717-728. doi: 10.1016/0028-3932(94)90031-0
- Moore, D. S., & McCabe, G. P. (2001). *Introduction to the Practice of Statistics*. New York: W. H. Freeman. 4ª Ed. 2003.
- Pryce, G. (2005). *Inference and Statistics in SPSS*. Glasgow: GeeBeeJey Publishing.
- Salguero, M. P. (2007). El procesamiento de los números arábigos: una aproximación desde la neuropsicología cognitiva. Huelva: Universidad de Huelva.
- Salguero, M. P., & Alameda, J. R. (2003). El procesamiento de los números y sus implicaciones educativas. XXI, *Revista de Educación*, *5*, 181-189.
- Salguero, M. P., & Alameda, J. R. (2007). *Batería para la Evaluación del Procesamiento Numérico y el Cálculo*. Huelva: Hooverand.
- Salguero, M. P., & Alameda, J. R. (2010). Diferencias neuroanatómicas y funcionales entre razonamiento numérico y cálculo. *Análisis y modificación de conducta*, *36*, 153-154, 33-42.
- Salguero, M. P., & Alameda, J. R. (2011). *Procesamiento Numérico y Cálculo. Pruebas para su evaluación*. Leipzig: Editorial Académica Española.
- Salguero, M. P., Lorca, J. A., & Alameda, J. R. (2003). Procesamiento numérico y cálculo: evidencia de un caso desde la Neuropsicología Cognitiva. *Revista de Neurología*, *36* (9), 817-820.
- Salguero, M. P., Lorca, J. A., & Alameda, J. R. (2004). Independencia funcional del conocimiento numérico léxico y la representación de la magnitud: Evidencia de un caso. *Revista de Neurología*, *39* (11), 1038-1042.
- Ungerleider, L. G., & Mishkin, M. (1982). Two cortical visual systems. In D. J. Ingle, M. A. Goodale & R. J. Mansfield (Eds.), *Analysis of visual behavior* (pp. 549-586). Cambridge, MA: MIT Press.
- Van Harskamp, N. J., & Cipolotti, L. (2001). Selective impairments for addition, subtraction and multiplication. Implications for the organisation of arithmetical facts. *Cortex*, *37*, (3), 363-388. doi: 10.1016/S0010-9452(08)70579-3

(Article received: 19-10-2011; reviewed: 16-10-2012, accepted: 16-10-2012)