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Number sense and academic buoyancy among middle school students: A serial mediation model of mathematical metacognition awareness and math anxiety

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Título: Sentido numérico y perseverancia académica entre estudiantes de secundaria: un modelo de mediación en serie de conciencia metacognitiva matemática y ansiedad matemática.

Resumen: El presente estudio examinó si el sentido numérico estaba relacionado con la perseverancia académica y si esta relación estaba mediada por la conciencia metacognitiva matemática y la ansiedad matemática. Se recopilaron datos de 231 estudiantes de séptimo y octavo grado de escuelas intermedias mediante medidas de autoinforme de sentido numérico, perseverancia académica, conciencia metacognitiva y ansiedad matemática. Los resultados revelaron una asociación positiva significativa entre el sentido numérico y la conciencia de metacognición matemática, así como una asociación negativa significativa entre el sentido numérico y la ansiedad matemática. Además, los resultados indicaron que el sentido numérico facilitó la perseverancia académica a través de la conciencia de la metacognición matemática y luego de la ansiedad matemática. Estos hallazgos contribuyen a la comprensión de cómo estos factores pueden afectar la perseverancia académica en los estudiantes de secundaria.

Palabras clave: Perseverancia académica. Sentido numérico. Ansiedad matemática. Conciencia metacognitiva matemática.

Abstract: The present study examined whether number sense was related to academic buoyancy and whether this relationship was mediated by mathematical metacognition awareness and mathematics anxiety. Data was gathered from 231 students in grade 7 and 8 middle schools through self-report measures of number sense, academic buoyancy, metacognition awareness, and math anxiety. The results revealed a significant positive association between number sense and mathematical metacognition awareness, as well as a significant negative association between number sense and math anxiety. Moreover, results indicated that number sense facilitated academic buoyancy via mathematical metacognition awareness and then math anxiety. These findings contribute to the understanding of how these factors may impact academic buoyancy in middle school students.

Keywords: Academic buoyancy. Number sense. Math anxiety. Mathematical metacognition awareness.

Introduction

Enhancing number sense in children is crucial for their ability to successfully navigate math-based tasks in their daily lives (Yang, 2019). However, according to a report by the Organization for Economic Co-operation and Development (OECD), a significant proportion of individuals exhibit low levels of mathematical literacy, struggling with tasks such as two-step calculations and understanding basic numerical concepts like fractions, decimals, and percentages across OECD participating countries (Kankaraš et al., 2016). These challenges begin to arise early in students' daily school lives, leading to academic difficulties such as poor performance, competing deadlines, exam pressure, and negative engagement with teachers (Kena et al., 2016). Since it is not easy to resist these issues, some students therefore experience long lasting negative effects including deep anxiety, difficulty learning, loss of motivation, and less interaction in class (Martin & Marsh, 2008). Academic buoyancy, described as the ability to effectively overcome academic setbacks and difficulties commonly faced during the academic life, is a key construct in understanding how students deal with these challenges (Martin & Marsh, 2006; 2008; 2009). The concept of academic resilience, which has been studied extensively in psychology, is closely related to academic buoyancy (Toprak-Çelen, 2020). Academic resilience is the ability to achieve success in school and other areas of life despite facing challenges and adversity due to early experiences, traits, and conditions (Wang et al., 1994, p.46). However, Martin and Marsh (2008) argue that academic buoyancy is a more specific construct of adversity that specifically refers to the common challenges and difficulties faced by students in their academic lives. The notion of academic buoyancy is served as a resolution to the challenge presented by the previous research on academic resilience that studied a broader range of school students (Martin & Marsh, 2008). According to Datu and Yuen (2018), there are two lines of evidence which highlight the positive association of academic buoyancy with key academic and psychological outcomes. First, the evidence clearly points to the beneficial impact of academic buoyancy on indicators of adaptive academic functioning. The second line of research suggests a negative association between academic buoyancy and certain maladaptive outcomes. Academic buoyancy specifically refers to the common experiences faced by students in their academic lives, such as isolated poor grades, 'patches' of poor performance, 'typical' stress levels, dips in motivation and engagement, daily pressures, and negative feedback on schoolwork (Martin & Marsh, 2008). This concept has significant merit, as it is relatable to many students, and identifying key predictors of academic buoyancy can provide valuable insights for educators to better support students in facing and overcoming academic difficulties. Thus, it is crucial to determine key predictors of academic buoyancy in order to assist educators in improving students' academic performance and well-being.

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Predictors of Academic Buoyancy

Some studies have looked into factors that may predict academic buoyancy. Previous longitudinal research has shown that self-efficacy, engagement, and control all predict future academic buoyancy (Martin & Marsh 2008; Martin 2009). In contrast, factors such as failure avoidance, anxiety, unclear control, emotional instability, neuroticism, and exam anxiety have all been linked to decreased academic buoyancy (Collie et al. 2017; Martin & Marsh 2008).

Collie et al. (2015) investigated whether academic buoyancy and student achievement had a direct or indirect relationship. The results demonstrated that there was a correlation between academic buoyancy and student achievement, but not a direct relationship. Additionally, the study found that time control was a factor that influenced the relationship between academic buoyancy and student achievement (Phase 1). Furthermore, over time, a cyclical process may occur amongst the three elements, academic buoyancy, time control and achievement (Phase 2). Previous research by Martin and Marsh (2008) has shown that academic buoyancy, or the ability to maintain a positive outlook and handle setbacks during academic pursuits, is linked to a variety of outcomes in adolescents, including homework completion and numeracy skills in relation to math performance. Additionally, Martin et al. (2010) have highlighted the importance of various motivational factors, including the ability to cope with challenges, in predicting academic buoyancy. Previous research has found that anxiety and uncertainty about control negatively predict academic buoyancy (Martin & Marsh, 2006, 2008; Martin et al., 2010).

Several research (Collie et al. 2015; Datu & Yang 2016; Martin, 2013; Putwain, et al., 2020; Putwain & Daly 2013) have also noted the potential correlation between academic buoyancy and a variety of favorable educational and psychological outcomes. While there is a significant link between academic buoyancy and academic achievement, the effect size is small (Martin, 2014). Martin and Marsh (2008) discovered that academic buoyancy had a significant effect on various academic outcomes in adolescents, including homework completion, absenteeism (in a negative way), and numeracy in relation to mathematics performance. Additionally, research suggests that academic buoyancy may be domain-specific, resulting in different predictions for areas such as mathematics and reading (Colmar et al., 2019). Marsh and colleagues have also shown that there is a strong correlation between academic performance in math and reading, and self-concept in those respective subjects, but no correlation between math and reading self-concepts. This suggests that there is domain-specificity in these relationships (Marsh & Yeung, 1997). Additionally, Colmar et al. (2019) found that self-concept plays a role in mediating the relationship between academic buoyancy and academic performance in primary school students, but this process differs for mathematics and reading. Umay et al. (2006) also investigated the mathematics curriculum according to the National Council of Teachers of Mathematics' (NCTM) principles and standards, and found that while the importance of number sense was emphasized in the curriculum, there was no learning outcome or activity in terms of developing number sense. These studies have shown that both number sense and academic buoyancy have positive relationships with math performance and academic (mathematics) achievement. Based on these studies, it can be concluded that number sense positively related to academic buoyancy

Number sense

The acquisition of number sense in children is widely recognized as a crucial aspect of mathematics education and curricula globally (Alkaş-Ulusoy, 2020; Cheung & Yang, 2018; Kayhan-Altay & Umay, 2013; Yang, 2019). According to education experts, number sense encompasses a range of components, including counting, numerical magnitude comparisons, estimation, number patterns, and the combination of quantities (Locuniak & Jordan, 2008; Shumway, 2016). Carpenter (1989) defined number sense as the ability to understand and utilize numbers and operations in a flexible manner that can be developed over time. Similarly, Yang (2019) defined number sense as the ability to handle problems faced in daily life using flexible and efficient strategies. Despite the varying definitions, researchers agree that number sense is a vital component of mathematics education. According to a study by Reyes (1986), 80% of mathematics used in daily life is based on numerical operations that require mental computation or estimation rather than paper-and-pencil activities (Reyes, 1986). Therefore, many studies have found number sense to correlate with estimation skills and mental computational abilities (Calvert, 1999; Sowder, 1992). The NCTM report (1989) described children with strong number sense as possessing the ability to understand the meanings of numbers, establish connections between numbers, recognize relative magnitude, comprehend the effects of mathematical operations, and determine appropriate measurement references for common objects and situations. The cultivation of number sense skills in preschoolers has been shown to have a positive impact on their mathematics learning in later years, while a lack of number sense can impede mathematics learning (Jordan et al., 2009).

According to Yang and Wu (2010), there are several reasons why the development of number sense is crucial for students' success in mathematics. According to Yang and Wu (2010), the development of number sense is crucial to a student's future success in mathematics education. This is because number sense requires a way of thinking that is characterized by flexibility, creativity, efficiency, and logic, which make understanding mathematical concepts at higher levels easier. Number sense also plays a crucial role in shaping adults' mathematical thinking and problem-solving abilities by providing understanding in real-world contexts. Yang and Wu (2010) also suggest that an overemphasis on written computation can impede students' development of number sense, which limits their mathematical thinking and understanding abilities. Alkaş-Ulusoy (2020) noted that it is important for students to internalize the concept of number sense and apply it when necessary rather than only demonstrating it when prompted.

Consequently, number sense refers to an individual's comprehension and flexibility in using numbers and mathematical operations. It is not directly taught as a specific topic but is developed through mathematical experience. The components of number sense include understanding the meanings of numbers and operations, using multiple representations of numbers and operations, recognizing the relative magnitude of numbers, being able to compose and decompose numbers flexibly, and judging the reasonableness of computational results through different strategies (Maghfirah &Mahmudi, 2018; Spinillo et al., 2021) These components contribute to mathematical understanding by enabling students to make sense of numbers, develop efficient strategies, solve numerical problems, and estimate results. Number sense helps students move beyond relying on rule-based methods and develop a deeper understanding of mathematical concepts (Chen et al., 2013). Teachers play a crucial role in promoting number sense through explicit instruction and using instructional strategies such as five frames, number lines, think aloud, and concrete to abstract representations (Baker, 2019). Developing number sense in students is important for their mathematical achievement and overall mathematical development (Faulkner & Cain, 2009).

Number sense and academic buoyancy

Number sense is a crucial aspect of mathematical reasoning, as it encompasses the ability to use numbers and operations flexibly and to develop effective strategies in mathematical problem-solving contexts (Yang, 1995). Recent research on academic buoyancy and math performance (e.g., Colmar et al., 2019; Marsh & Yeung, 1997) has highlighted the importance of number sense as a predictor of academic performance. Many researchers (Markovits & Sowder, 1994; McIntosh et al., 1992; NCTM, 2000; Reys & Yang, 1998; Sowder, 1992; Yang, 2003; Yang, Hsu & Huang, 2004; Yang, Li & Li, 2008) have pointed out that, number sense is characterized by flexibility, inventiveness, efficiency, and reasonableness in the use of numbers and operations. When solving mathematical problems, an individual with strong number sense is able to approach the task from a sense-making perspective, plan and control their actions, and have flexibility and reasonable judgment in determining the appropriateness of their solution (Mohamed & Johnny, 2010).

Previous research has consistently shown a positive correlation between number sense and various mathematical abilities, including estimation, mental computation, and problem solving (Louange & Bana, 2010; Pike & Forrester, 1997; Tsao, 2004; Yang & Huang, 2004). The development of strong number sense skills enables children to employ versatile and efficient problem-solving strategies (Yang & Wu, 2010), which can subsequently reduce math anxiety and enhance academic buoyancy, allowing them to better navigate academic challenges. Given these findings, we feel there is support for the importance of examining academic buoyancy as it relates to number sense.

Metacognition and mathematical metacognition awareness

The concept of metacognition, as defined by Flavell (1976), refers to an individual's knowledge and active regulation of their own cognitive processes. Research has repeatedly demonstrated the significance of metacognition in enhancing problem-solving performance in mathematics (Artz & Armour-Thomas, 1992; Asık, 2009; Asık & Erktin, 2019; Desoete & Veenman, 2006; Mihalca et al., 2017). Metacognition, which is seen as higher-order thinking, and number sense are both considered crucial to successful mathematics education. Metacognition is the understanding and control of one's own thinking processes and plays an important role in mathematical problem-solving (Çekirdekçi et al., 2018). Most mathematics curricula place emphasis on both metacognition and number sense, and multiple studies have found a significant relationship between math achievement with number sense (Çekirdekci et al., 2018; Jordan, 2010) and metacognition (Aşık & Sevimli, 2015; Memiş & Arcan, 2013). Research has also demonstrated a relationship between number sense and metacognition (Çekirdekci et al., 2018; Shumway, 2016). Thus, it is important to investigate the connection between these two concepts, as they are both essential for mathematical thinking and reasoning.

Mathematical metacognition awareness refers to the ability of individuals to have knowledge about their own thinking processes and cognition in mathematical reasoning (Tak et al., 2023). It involves the use of metacognitive strategies such as planning, monitoring, and evaluating in solving mathematical problems (Tak et al., 2022a). Research has shown that metacognitive awareness is positively related to mathematical reasoning abilities among university students (El Walida, & Sa'dijah,. 2022). It has been found that students with higher metacognitive awareness tend to have better mathematical problem posing self-efficacy and achievement (Celik & Arslan, 2022). Metacognitive awareness is also associated with the utilization of strategies and the identification of known aspects and contradictions in mathematical problem-solving (Tak et al., 2022b). The development of metacognitive strategies and the application of metacognitive awareness in mathematics learning models can facilitate students in solving controversial mathematical problems and improving their mathematical reasoning abilities.

Math anxiety

According to the findings of the study conducted by Namkung et al., (2023), math anxiety is represented by both cognitive and affective dimensions in both males and females, and math anxiety has a detrimental impact on students' mathematical performance. Math anxiety has been found to be consistently associated with low math performance, with a negative correlation between math anxiety and math performance, $r = -.34$, 95% CI [−.37, −.31] (Namkung et al., 2019 for meta-analyses). Mathematics anxiety refers to feelings of tension and anxiety experienced by individuals when solving mathematical problems, and it can impede the ability to manipulate numerical data in both academic and daily life (Richardson & Suinn, 1972). It is distinct from other forms of anxiety in that it pertains specifically to math-related situations (Carey et al., 2017). Additionally, math anxiety may be distinct from, but related to, general and test anxiety (Ashcraft et al., 2007). Few studies have explored the relationship between math anxiety and number sense. According to Ak and Ertekin (2020), there is a moderate negative correlation between number sense and math anxiety, with increasing number sense resulting in a decrease in math anxiety levels. Furthermore, in a study conducted by Maldonado-Moscoso et al. (2020), found that math anxiety is an intermediary factor in the link between math abilities and numerosity perception (approximate number system; ANS acuity) in individuals with high mathematical anxiety. Moreover, some researchers found that the precision of numerosity perception (also called numerical acuity) correlates with math performance, with individuals with better math skills also performing more precisely in the numerosity tasks (Anobile, et al., 2018; Mazzocco, et al., 2011). In this study, number sense was examined under the components (understanding the meanings of numbers and operations; recognizing relative number size; being able to compose and decompose of numbers; recognizing the relative effect of operations on numbers; judging the reasonableness of computational results) defined by Yang (1995).

The study aimed to examine the impact of math anxiety on math skills and numerosity perception, and the results indicated that individuals with high levels of math anxiety tend to have a less precise approximate number sense. However, it has been suggested that math anxiety may stem from low numerical abilities that hinder progress in mathematical problem solving (Maloney, 2016). Academic challenges and setbacks may also contribute to negative feelings and tension around numeracy tasks, leading to increased math anxiety (Putwain et al., 2012). The relationship between academic buoyancy, number sense, and math anxiety remains unclear, and further research is needed to investigate the potential mechanisms of this association.

The present study

Many students face difficulties in mathematics, which can negatively impact their academic buoyancy, as per the research conducted by Martin and Marsh (2008). As a result, the current study focuses on mathematics. In line with the theoretical explanations mentioned above, this study focuses on the indicators of academic buoyancy in middle school students. The present study investigated the associations between number sense with metacognitive awareness. At the same time, the study investigates the relationship between math anxiety with academic buoyancy. Consequently, the following hypotheses were proposed in the present study:

H_1: *Mathematical metacognition awareness mediated the relationship between number sense and academic buoyancy,*

H_2: *Math anxiety mediated the relationship between number sense and academic buoyancy.*

H_3: *The association between number sense and academic buoyancy was serially mediated by both mathematical metacognition awareness and math anxiety.*

Method

Participants and procedure

Our sample comprises 231 students from randomly selected eight middle schools in a small-sized city in the northern east part of Türkiye. In this sample, there were 130 girls (56.3%) and 101 boys (43.7). The sample consists of seventh grade (n = 127, 55%) and eight grade (n = 104, 45%). The average age of the participants was 13.45 (SD = 0.49) years. Data were collected via an anonymous form through a paper-pencil self-report questionnaire in a 40-minute class at school. Before they could participate in the survey, they stated that they were volunteers and gave informed consent. Ethics approval was granted by the University Scientific Research and Ethical Review Board. The research procedures were carried out in accordance with the declaration of Helsinki.

Measures

The Modified Abbreviated Mathematics Anxiety Scale. The Modified Abbreviated Mathematics Anxiety Scale (mAMAS; Carey et al., 2017) is a revised version of the AMAS (Hopko et al., 2003), which is a widely used psychometric instrument for measuring mathematics anxiety among undergraduate students. The mAMAS (Carey et al., 2017) is a self-report instrument that comprises nine items measuring Learning Math Anxiety (5 items) and Math Evaluation Anxiety (4 items), each rated on a 5-point Likert scale, ranging from low to high anxiety. The present study used the Turkish mAMAS (Kul, et al., 2024). The Turkish mAMAS has very good construct validity and acceptable reliability (α = .82). This study results indicate that the measurement used to measure Turkish students' MA is reliable, as evidenced by the meaningful correlations between its subscales and the replicating factor structure of the original scale. Therefore, the Turkish version of the mAMAS is a valid and reliable tool for assessing students' MA. In this study, the mAMAS also demonstrated very good reliability (α=.84). In addition, Cronbach's alpha indicated the learning math anxiety dimension at .73 and the math evaluation anxiety dimension at .72.

Academic Buoyancy Scale. Academic Buoyancy Scale (ABS) was created by Martin and Marsh (2008). The Cronbach's α coefficient for Time 1 and Time 2 was respectively .80 and .82. Martin et al. (2017) used the ABS have found it to be valid and reliable in various cultural contexts. For academic buoyancy, we used a translated version of a common questionnaire by Martin and Marsh (2008). For Turkish translation, the ABS, originally in English, was translated using a forward-translation approach by two professional bilingual translators with expertise in psychology and math education. Any minor discrepancies in the translation were addressed and resolved through mutual agreement. A bilingual expert translator performed back translation to compare it to the original text. The scale consisted of four items referring to subject–independent academic buoyancy (e.g., "I'm good at dealing with setbacks – e.g.,." I don't let a bad mark affect my confidence"). Again, internal consistency was satisfactory in this study (α=.75). Hence, the Turkish version of the ABS is a valid and reliable tool for assessing Turkish students' buoyancy.

Number Sense Scale (NSS). Number Sense Scale (NSS) was developed by Kayhan-Altay and Umay (2013) to measure the number sense abilities of middle school students. The NSS was originally developed in Turkish language. The scale includes 17 questions that focus on six key components of number sense, such as understanding number meanings, decomposing and recomposing numbers, recognizing number magnitude, and using reference points proposed by Yang (1995). Students are awarded 1 point if they solve a problem using number sense, regardless of whether the answer is correct or not. If a student uses calculation or standard formulas or gives an incorrect answer, they are awarded zero points. The NSS is divided into three sub-scales: "Flexibility in calculation (8 items)," "Conceptual thinking in fractions (4 items)" and "Use of reference points (5 items)". The researchers (2013) reported a reliability coefficient of 0.86 for the NSS. In this study, the internal consistency of the scale was determined to be .83 using Cronbach's alpha. In this study, Cronbach's alpha showed the flexibility in calculation sub-scale at .72, the conceptual thinking in fraction sub-scale at .61, and the use of reference points dimension at .69. Some sample items related to the scale are given below. Sample items in the next Figure 1.

Figure 1
Sample items related to the Number Sense Scale

Sub-scale 1: Flexibility in calculation :
Is the result of 6464 × 0.54 greater than 3232, or is it smaller? Why?

Sub-sclae 2: Conceptual thinking in fractions:
Which letter on the number line corresponds to a fraction where the numerator is only slightly larger than the denominator? Explain how you found it.

Sub-scale 3: Use of reference points:
Which sum is greater than 1? Explain your reasoning.

a. $\frac{5}{11}+\frac{3}{7}$ *b.* $\frac{7}{15}+\frac{5}{12}$ *c.* $\frac{1}{2}+\frac{4}{9}$ *d.* $\frac{5}{9}+\frac{8}{15}$

Mathematical Metacognition Awareness Scale (MMAS). Mathematical Metacognition Awareness Scale (MMAS) was developed by Kaplan and Duran (2016) to assess the mathematical metacognition awareness perception levels of middle school students. The scale, which has 23 items in total, has three dimensions, and the scale items are scored on a 5-point scale (1=Never; 5=Always). The first factor, consisting of 8 items, was named "mathematical knowledge," the second factor, consisting of 8 items, was named "mathematical monitoring," and the final factor, consisting of 7 items, was named "mathematical determination." Total reliability of the scale is .90. In the present study, the construct validity of the scale was provided and the Cronbach alpha internal consistency value was .93 in the reliability analysis. In addition, Cronbach's alpha showed the mathematical knowledge sub-scale at .86, the mathematical monitoring sub-scale at .82, and the mathematical determination dimension at .79.

Statistical analysis

This study examined the relations between number sense, mathematical metacognition awareness, math anxiety, and academic buoyancy. First, descriptive statistics and correlations among the study variables were calculated. Then, this study was tested whether number sense on academic buoyancy serially mediated by mathematical metacognition awareness and math anxiety via a two-step structural equation modelling (SEM). We first ran a measurement model to identify four latent variables, representing the indicators. We then ran a structural model to test the direct and indirect associations between variables. The adjustment of the examined SEM to empirical data was investigated recurring to well-known goodness of fit indicators: relative chi-square (χ2/df), with a cut-point of ≤ 3 indicating a good fit (Kline, 2023); the Comparative Fit Index (CFI), where values ≥ 0.95 suggest excellent model fit (Hu & Bentler, 1999); Bollen's Incremental Fit Index (IFI), also recommending a threshold of ≥ 0.95 for acceptable fit (Bollen, 1989); Normed Fit Index (NFI), with a preferred cut-point of ≥ 0.90 (Bentler & Bonett, 1980); the Tucker-Lewis Index (TLI), where values close to 1, typically ≥ 0.95, indicate a good fit (Tucker & Lewis, 1973); Goodness of Fit Index (GFI), with ≥ 0.90 as an acceptable level (Jöreskog & Sörbom, 1986); the Standardized Root Mean Residual (SRMR), ideal when ≤ 0.08 (Hu & Bentler, 1999); and the Root Mean Squared Error of Approximation (RMSEA), where ≤ 0.06 indicates a good fit and values as high as 0.08 are acceptable in some contexts (Steiger, 1990). Lastly, we performed a bootstrapping procedure with 10,000 iterations in order to calculate confidence interval calculation of the effects.

Results

Descriptive statistics

The results of the descriptive statistics, reliability coefficients and correlation analysis are shown in Table 1. Correlation analyses revealed that number sense positively related to mathematical metacognition awareness (r = 0.434, p < .001) and academic buoyancy (r = 0.408, p < .001) and negatively related to math anxiety (r = -0.464, p < .001). Mathematical metacognition awareness was positively associated with academic buoyancy (r = 0.539, p < .001), negatively associated with mat anxiety (r = −0.544, p < .001). Lastly, math anxiety negatively related to academic buoyancy (r = −0.536, p < .001).

Table 1
Descriptive statistics, reliabilities and correlations for the study variables

Variable	*M*	SD	Skewness	Kurtosis	α	ω	1	2	3	4
1. Number sense	4.66	3.82	.900	-.051	.829	.830	–			
2. Metacognition awareness	76.01	18.69	-.402	-.345	.931	.932	.434**	–		
3. Math anxiety	25.26	8.11	.158	-.880	.837	.838	-.464**	-.544**	–	
4. Academic buoyancy	12.32	3.85	-.070	-.722	.750	.752	.408**	.539**	-.536**	–

Note. **p < .001

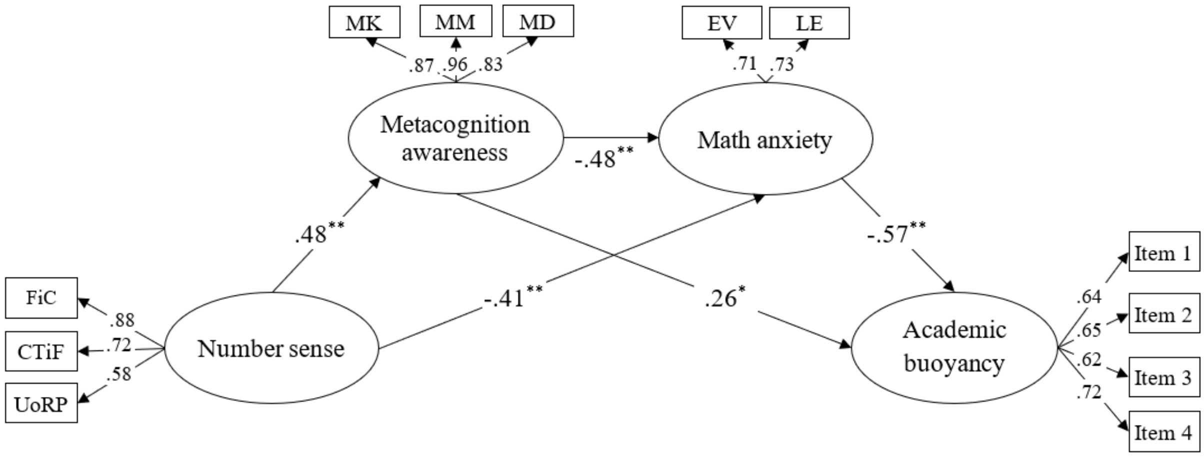
Structural equation modelling

First, we determine if the latent constructs are appropriate measures by testing the measurement model. The measurement model consisted of four latent variables (number sense, mathematical metacognition awareness, math anxiety, and academic buoyancy) and 12 observed variables. The measurement model was found to be an acceptable fit to the data: CMIN/df = 1.95, CFI = 0.96, IFI = 0.96, NFI = 0.93, TLI = 0.95, GFI = 0.94, SRMR = 0.039, RMSEA = 0.06. All factor loadings of the measurement model as indicators were significant, ps < .001.

After confirmed the measurement model, serial mediation models were examined. We tested the effect of number sense on academic buoyancy through serial mediation of mathematical metacognition awareness and math anxiety in two different structural models. In the first structural model (fully mediated) was examined without the direct effect path from the independent variable (number sense) to dependent variable (academic buoyancy). The fully mediated model fits the observed data acceptable: CMIN/df = 1.91, CFI = 0.96, IFI = 0.97, NFI = 0.93, TLI = 0.95, GFI = 0.94, SRMR = 0.039, RMSEA = 0.063, AIC = 151.619, ECVI = 0.65. In the second structural model (partially mediated), the path from the independent variable to the dependent variable is added. The partially mediated model showed good fit to the data: CMIN/df = 1.94, CFI = 0.96, IFI = 0.96, NFI = 0.93, TLI = 0.95, GFI = 0.94, SRMR = 0.039, RMSEA = 0.064, AIC = 153.51, ECVI = 0.67. However, the path added from the independent variable to the dependent variable was not significant (β = .060, p>.05). In addition, there was not an improvement in model fit resulted from adding direct path to the model ($\Delta\chi^2$ = 0.11, df = 1, p>.05). The fully mediated model has also been the preferred model because of its lower AIC and ECVIs. The standardized factor loading of the fully mediated model are presented in Figure 2.

The results (see Figure 2) indicated that number sense significantly predicted mathematical metacognition awareness (β = .48, p < .01) and math anxiety (β = -.41, p < .01). Mathematical metacognition awareness significantly predicted math anxiety (β = -.48, p < .01). Mathematical metacognition awareness (β = .26, p < .05) and math anxiety (β = -.57, p < .01) were also found to have a significant effect on academic buoyancy. Lastly, bootstrapping analysis revealed that that number sense facilitated and academic buoyancy via mathematical metacognition awareness and then math anxiety (i.e., serial mediating effect), β = .489, SE = .066, 95% CI = [.364, .620].

Figure 2
Standardized factor loadings for the serial model.



Note. $^{*}p < .05$, $^{**}p < .01$, *MK:* mathematical knowledge, *MM*: mathematical monitoring, *MD*: mathematical determination, *EV*: evaluation, *LE*: learning, *FiC*: flexibility in calculation, *CTiF*: conceptual thinking in fractions, *UoRP:* use of reference points.

Discussion

In the present study, the relationship between number sense and academic buoyancy was examined in light of the potential mediating roles of metacognition awareness and mathematics anxiety. Through the analysis of data, it was found that the hypotheses were supported. Within this framework, the key findings were: (1) mathematical metacognition awareness mediated the relationship between number sense and academic buoyancy, (2) math anxiety mediated the relationship between number sense and academic buoyancy, and (3) the association between number sense and academic buoyancy was serially mediated by both metacognition awareness and math anxiety. As a result, the study's hypotheses were validated. These findings contribute to the understanding of the mechanisms underlying the relationship between these factors in middle school students.

The number sense construct is considered a fundamental aspect of students' mathematical learning and development by both education and cognitive psychology perspectives (Shumway, 2016). Number sense, along with metacognition, which is a form of reasoning, is essential for mathematical thinking and problem-solving (Çekirdekci et al., 2018; Ilko, 2021). Previous research has established that students who lack adequate number sense are unable to effectively demonstrate metacognitive skills, which involve developing strategies and reasoning through problems (Çekirdekci et al., 2018). In the current study, it was observed that an increased number sense was associated with increased metacognition awareness, and this was found to predict high levels of academic buoyancy. Flexible thinking, which is a component of number sense and is dependent on meaningful learning, is also a necessary thinking skill for problem-solving strategies in metacognition (Çekirdekci et al., 2018; Kaplan & Duran, 2016). Developing mathematical metacognition awareness is essential for becoming a proficient problem solver and a successful learner in mathematics. It helps individuals become more independent and effective learners by enabling them to take control of their own learning processes, identify areas where they need improvement, and develop strategies to overcome challenges. Additionally, several studies have established a link between metacognitive strategies and academic achievement (Asık, 2009; Eme & Rouet, 2001; Kitsansas, 2002). Furthermore, research by Miller et al. (2013) and Martin and Marsh (2009) has shown a relationship between academic buoyancy and achievement. Therefore, it can be inferred that high levels of metacognitive awareness can lead to increased academic buoyancy. The findings of this study are in line with theoretical perspectives. The construct of number sense, as defined by Howden (1989), encompasses the ability to recognize multiple paths to a solution and to form logical conclusions, rather than solely relying on rules. NCTM (1989) further elaborates that students with good number sense possess well-understood number meanings, have developed multiple relationships among numbers, recognize the relative magnitude of numbers, understand the effect of mathematical operations on numbers, and have developed referents for measuring common objects and situations in their environment. Maloney et al. (2011), proposed that having poor numerical/spatial skills could jeopardize the proper development of mathematical procedures, eventually contributing to the development of math anxiety. Teaching number sense encourages students to adopt flexible and efficient problem-solving strategies (Yang & Wu, 2010) and can also reduce their math anxiety as they develop confidence in their ability to solve problems. The current study supports this notion and confirms the second hypothesis, which posits that an increase in number sense performance is associated with a decrease in math anxiety levels. This finding is consistent with previous research that has established a negative correlation between number sense and math anxiety (Ak & Ertekin, 2020). Similarly, in the present study, math anxiety negatively related to academic buoyancy. High levels of math anxiety can lead to negative emotions such as anxiety, hopelessness and shame, which in turn predict difficulties in coping with math and lower achievement. Academic buoyancy, on the other hand, acts as a protective factor against the negative impact of test anxiety on academic performance, influencing self-regulative processes and enabling better examination performance (Putwain et al., 2016). Academic buoyancy impacts many aspects of teaching and learning, including student achievement, motivation, self-efficacy, self-esteem, classroom enjoyment, self-regulation strategies, stress and anxiety reduction, increased class participation, and test performance (Putwain et al., 2015). Therefore, fostering academic buoyancy may help mitigate the detrimental effects of math anxiety on academic success.

Finally, the third hypothesis of the current study was confirmed. In this framework, the study found that an increase in number sense is associated with increased metacognitive awareness, which in turn is associated with decreased math anxiety. Legg and Locker (2009) found that, in regard to the relationship between metacognition and anxiety, the results would suggest that individuals with higher anxiety benefit from higher levels of metacognition, as their math performance was similar to those individuals with low math anxiety.

Both metacognitive awareness and math anxiety were found to predict academic buoyancy. To the best of the researcher's knowledge, this is the first study to examine this specific relationship, making the results of this study particularly valuable in understanding the mechanisms by which number sense influences academic buoyancy. In this study, the serial mediator role of metacognitive awareness and math anxiety in the effect of number sense on academic buoyancy was investigated, and it was revealed that the findings supported serial mediation.

Implications

The present study indicated that number sense had a direct effect on academic buoyancy and that the relationship between number sense and academic buoyancy was serially mediated by both metacognition awareness and math anxiety. These findings highlight the importance of addressing and promoting number sense, metacognition awareness, and reducing math anxiety to enhance academic buoyancy among middle school students. The findings of the study have implications for educational practitioners and researchers in the field of mathematics education, providing insights on how to support students' academic buoyancy by addressing number sense, metacognition, and math anxiety. Ultimately, educators should concentrate on students' performance and misconceptions about number sense, which can be explored in depth using qualitative research methods and other instruments (for example, in-depth interviews, open-ended examinations, and a two/three-tier test). Academic buoyancy, luckily, is a skill that can be learned and improved (Martin, 2013b; Putwain et al., 2019). The 5Cs (confidence, coordination, commitment, control, and composure; see Martin et al., 2010) that support academic buoyancy are all accessible to relatively simple interventions (e.g., McInerney et al., 1997; O'Mara et al., 2006). Educational practitioners should focus on interventions that improve the academic buoyancy and lower math anxiety level to increase number sense and metacognition awareness in students. Mathematical games are an excellent example of such an intervention program since they allow students with varying math ability to engage and maybe learn from one another (Sarı & Szczygiel, 2022). It reveals the need for number sense to be included more in mathematics programs and to focus more on number sense strategies (mental processing, guessing, etc.). For this reason, universities that train teachers should include more emphasis on number sense in mathematics teaching; It may be useful to provide training explaining its effect on mathematics achievement and mathematics anxiety. In developing number sense in students, it is important that teachers, who are the implementers of curriculum, include number sense strategies in the learning environment.

Limitations

This study has several limitations that should be considered. Firstly, the research data was collected through self-report measures, which may have resulted in common method biases. Secondly, the cross-sectional design of the study does not allow for a clear understanding of the cause-effect relationship between the variables under investigation. Therefore, future studies using experimental and longitudinal designs are recommended to further explore the relationship between number sense, metacognition, math anxiety, and academic buoyancy.

Practitioners' points

- Strengthen number sense, metacognition awareness, and reducing math anxiety to enhance academic buoyancy among middle school students.
- The association between number sense and academic buoyancy was serially mediated by both metacognition awareness and math anxiety.
- Increased number sense was associated with increased metacognition awareness, and this was found to predict high levels of academic buoyancy, which in turn is associated with decreased math anxiety.

Conclusion

The present study provides evidence for the association between number sense, metacognition awareness, math anxiety, and academic buoyancy. Results indicate that an enhancement in number sense leads to increased metacognition awareness, which in turn leads to decreased math anxiety, resulting in higher levels of academic buoyancy. To promote our children' development of number sense, "practice in applying metacognitive strategies should be carried out by teachers of all content areas (subjects) beginning in the primary years" (Djudin, 2017). These findings suggest that educators and scholars in the field of education should prioritize the development of number sense and metacognition awareness in students to improve academic buoyancy. The study serves as a valuable guide for mathematics educators and scholars in the field of education to understand the importance of number sense and metacognition awareness in fostering academic buoyancy. The current study was quantitative, and further qualitative research is needed to determine how number sense enhances metacognition awareness and so promotes academic buoyancy. Future research should evaluate the current study's connections longitudinally to examine the directions of the associations and how such associations evolve over time.

Complementary information

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Ethical Approval.- All procedures in the study are in accordance with the ethical standards of the responsible human experiments committee and the 1975 Declaration of Helsinki, which was revised in 2000.

Conflict of interests.- The authors declare that they have no conflicts of interest.

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Appendix

Correlations and descriptive statistics for all indicators

	Descriptive				Bivariate correlations										
Variable	*M*	SD	Skewness	Kurtosis	1	2	3	4	5	6	7	8	9	10	11
1. FiC	2.32	2.06	.75	-.52	–										
2. CTiF	1.34	1.32	.64	-.75	.62**	–									
3. UoRP	1.00	1.20	1.42	1.78	.52**	.39**	–								
4. MK	28.58	7.36	-.66	-.33	.39**	.41**	.25**	–							
5. MM	25.80	6.76	-.23	-.51	.38**	.35**	.28**	.83**	–						
6. MD	21.64	6.09	-.13	-.54	.33**	.29**	.25**	.72**	.80**	–					
7. EV	14.10	4.33	-.46	-.81	-.45**	-.41**	-.26**	-.39**	-.44**	.33**	–				
8. LE	11.16	4.97	.78	-.44	-.35**	-.29**	-.21**	-.58**	-.47**	-.37**	.52**	–			
9. AB Item 1	2.94	1.19	-.05	-.73	.27**	.24**	.27**	.34**	.36**	.36**	-.31**	-.30**	–		
10. AB Item 2	3.21	1.29	-.22	-1.08	.28**	.28**	.18**	.35**	.38**	.31**	-.37**	-.46**	.40**	–	
11. AB Item 3	3.18	1.22	-.24	-.76	.28**	.21**	.22**	.39**	.46**	.39**	-.27**	-.35**	.40**	.38**	–
12. AB Item 4	3.00	1.39	.06	-1.28	.29**	.23**	.29**	.37**	.45**	.39**	-.37**	-.38**	.50**	.46**	.43**

Note. ** $p < .01$; *FiC*: flexibility in calculation, *CTiF*: conceptual thinking in fractions, *UoRP:* use of reference points, *MK:* mathematical knowledge, *MM*: mathematical monitoring, *MD*: mathematical determination, *EV*: evaluation, *LE*: learning, *AB*: academic buoyancy