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Analysing Strategy Use in Terms of the Four Parameters of Strategic Competence: Contributions from a Numerosity Judgement Task*

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Abstract: The present paper discusses three studies that all relied on Lemaire and Siegler's (1995) theoretical framework for analyzing people's strategy use in the context of a numerosity judgment task. This framework distinguishes between four different parameters of strategic competence: repertoire, frequency, efficiency and adaptivity. In Study 1, we applied this framework for studying developmental changes in children's numerosity judgment strategies. In Study 2, we analysed the contribution of intelligence on these four parameters and assessed the extent to which the provision of feedback could lead to an improvement in one or more of these parameters. In Study 3, we investigated whether variations in the task context would lead to alterations in one or several of these parameters. At the end of the paper, we reflect on the broader applicability of this theoretical framework and the research findings that it has yielded.
Key words: Conceptual knowledge; procedural knowledge; metacognitive skills; strategic knowledge; quantification tasks.

Título: Analizando el uso de estrategias en términos de cuatro parámetros de la competencia estratégica: Contribuciones de una tarea de juicio de cantidad.
Resumen: En el presente trabajo se analizan tres estudios basados en el marco teórico de Lemaire y Siegler (1995) para el análisis del uso de estrategias en el contexto de una tarea de juicio de cantidad (*numerosity*). Este marco distingue entre cuatro diferentes parámetros de la competencia estratégica: el repertorio, la frecuencia, la eficiencia y la adaptabilidad. En el Estudio 1, se aplicó este marco para el análisis de los cambios evolutivos en las estrategias sobre juicios de cantidad. En el Estudio 2, se analizó la contribución de la inteligencia en estos cuatro parámetros y se evaluó en qué medida el suministro de información podría conducir a una mejora en uno o más de estos parámetros. En el Estudio 3, se investigó si las variaciones en el contexto de la tarea darían lugar a alteraciones en uno o varios de estos parámetros. Al final de este trabajo, reflejamos la amplia aplicabilidad de este marco teórico y los resultados que la investigación ha proporcionado.
Palabras clave: Conocimiento procedimental; conocimiento conceptual; habilidades metacognitivas; conocimiento estratégico; tareas de cuantificación.

In the last 25 years, numerous studies have shown that individuals of different ages exhibit a remarkable variability in their strategies for accomplishing various cognitive tasks. This multiple strategy use is not only present in arithmetic, the domain in which it was initially observed (Cooney, Swanson, & Lad, 1988; Geary & Wiley, 1991; Lemaire, Arnoud, & Lecacheur, 2004; Siegler, 1986), but also in other domains of human cognition such as scientific reasoning (Kuhn, Schauble, & Garcia-Milla, 1992), spelling (Rittle-Johnson & Siegler, 1999), decision making (Payne, Bettman, & Johnson, 1988), time telling (Siegler & McGilly, 1989), serial recall (McGilly & Siegler, 1990), currency conversion (Lemaire & Lecacheur, 2001), etc. The different strategies in one's repertoire typically vary in their respective speed and accuracy across problems. As such, they allow individuals to optimize their performance by adapting their strategy choices to the different problem characteristics and to the situational demands.

A useful framework for analyzing people's strategy use in a particular task has been provided by Lemaire and Siegler (1995). These authors distinguish between four different parameters of strategic competence: (a) the strategy *repertoire* which refers to the set of strategies an individual uses to accomplish a particular task, (b) the *frequency* of strategy use indicating how often the different strategies that are present in an individual's repertoire are selected, (c) the *efficiency* of strategy execution which specifies the ease (*i.e.*, the speed and/or accuracy) with which a strategy is applied and, (d) the *adaptivity* of strategy choices which refers to the extent to which an individual calibrates his/her strategy choices as a function of the problem and strategy characteristics. This framework has initially been developed to investigate developmental changes in children's strategy use. According to this perspective changes in any of the parameters of strategic competence can lead to an improvement in speed and accuracy of overall task performance. Indeed, children can improve their performance in a specific task by: (a) the acquisition of new, more advanced strategies and the abandonment of older less efficient ones, (b) an increasing reliance on more advanced strategies and a less frequent use of less efficient strategies, (c) an improvement in the speed and accuracy with which each of the different strategies is executed, and (d) an increasingly precise fit of the strategy choices to the demands of the problems and the limits of one's own performance.

Siegler and Lemaire (1997) devised a method which enabled a proper assessment of all parameters of strategic competence, namely the *choice/no-choice method*. It involves testing each participant under two types of conditions: (a) a *choice* condition in which participants can freely choose

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which strategy to use on each problem of the task, and (b) several *no-choice* conditions in which participants are required to use a given strategy on all problems. Ideally, the number of no-choice conditions equals the number of (pivotal) strategies in the choice condition.

Until recently, the large majority of studies on the selection and use of cognitive strategies in a wide array of tasks relied solely on the choice method. This method has, however, a serious drawback since it leads to biased measures of task performance due to selection effects. These selection effects are caused by the distinct strategies being used unequally often on the different types of problems as well as by individual differences in strategy preferences. A strategy that is used mainly to solve easy problems, or primarily applied by the most able subjects, will seem more efficient than a strategy that is almost exclusively applied to solve the most difficult problems, or employed most frequently by the least skilled subjects. A proper investigation of participants' strategy adaptivity, however, requires unbiased estimates of strategy performance that can function as a criterion with which the actual strategy use (as measured in the choice condition) can be compared.

Such unbiased estimates of a strategy's performance characteristics can be obtained by applying the no-conditions by requiring participants to use a given strategy on all trials in the no-choice conditions. As such, this method provides data for a proper examination of all four parameters of strategic competence. First, the choice condition yields information about participants' strategy repertoire as well as about the frequency of strategy use. Second, the no-choice conditions provide unbiased estimates of the performance characteristics of the different strategies under consideration. Finally, a comparison of choice and no-choice data enables assessing participants' strategy adaptivity by examining whether participants prefer one strategy above another on a particular set of problems because it is faster and/or more accurate on that set of problems.

The choice/no-choice method has already been successfully applied in the context of adults' multiplication (Siegler & Lemaire, 1997), young children's addition and subtraction (Torbeyns, Verschaffel, & Ghesquière, 2004), younger and older adults' currency conversion (Lemaire & Lecacheur, 2001), adults' computational estimation (Imbo, Duverne, & Lemaire, 2007), young children's spelling (Lemaire & Lecacheur, 2002a), and adults' reasoning (Dierckx, Vandierendonck, & Pandelaere, 2003).

In the present paper, we will discuss three studies in which we used Lemaire and Siegler's (1995) framework to investigate several aspects of people's strategic behaviour in the context of a numerosity judgment task. Moreover, two of these studies also implemented the choice/no-choice method for assessing the four parameters of strategy competence in this context. The line of research presented here will demonstrate that Lemaire and Siegler's framework has a much broader applicability than for which it was originally intended, namely the examination of developmental changes in children's strategies in cognitive tasks. More specifically, we will show that, besides addressing developmental issues, this framework can also be applied to examine the influence of other subject variables than age as well as the impact of contextual variables on people's strategy use. Before discussing these studies, we will provide some general information about the paradigm that was used in all these studies, followed by a rational task analysis. Next, we will discuss the method that we used for identifying participants' solution strategies.

Paradigm and Rational Task Analysis

In all our studies we used a task in which participants had determine different numerosities of coloured blocks that were presented in a grid structure (see Figure 1). It is claimed that the appropriate completion of this task requires a complex interplay of numerical, computational, and metacognitive knowledge and skills. Based on a rational task analysis, we distinguish between two main strategies for accomplishing this task: an addition strategy and a subtraction strategy. The choice for one of these strategies depends, among other things, on the ratio of coloured blocks to empty squares in the grid. When there are relatively few blocks in the grid, we expect that people will select the *addition strategy* by means of which the total quantity of blocks in the grid is divided into a number of subgroups, the numerosity of blocks in each subgroup is determined and this result is added to a running total. When there are many blocks and few empty squares in the grid, the *subtraction strategy* is expected to be carried out. This strategy consists of subtracting the determined number of empty squares (via an addition strategy) from the (estimated or computed) total number of squares in the grid.

According to our rational task analysis, the choice for and the efficiency of the subtraction strategy will depend on participants' mastery of several knowledge elements and abilities that are required for a proper application of this strategy. First of all, users of the subtraction strategy must realize that the grid size can (easily) be calculated by multiplying the number of rows and the number of columns in the grid. Second, the product of this multiplication must be automatised, so that it can be retrieved quickly and accurately from long-term memory. Third, they must master a procedure for mentally calculating subtractions. Finally, they must be able to identify for themselves the point at which the addition strategy is replaced by the subtraction strategy (*i.e.*, the so-called '*change point*'). It is important to note here that a strategy must not necessarily be rationally chosen or consciously executed. It can be selected and executed without involving consciousness.

Figure 1: Examples of a trial with respectively 20 and 80 coloured blocks in a 10 x 10 grid.

Both the addition and subtraction strategy are assumed to lead to a specific pattern of response times (and error rates) as a function of the numerosity of blocks that is present in the grid. The application of the addition strategy is assumed to lead to linearly increasing response times (and error rates) with an augmenting numerosity of blocks. Indeed, the more blocks in the grid, the more time required to count them. The use of the subtraction strategy, on the contrary, will lead to linearly decreasing response times (and error rates) with an increasing numerosity of blocks. The more blocks that are present in the grid, the less empty squares that need to be counted and, thus, the less time is needed to execute the counting procedure. Thus, when solely the addition strategy is used, one can expect a linearly increasing response-time pattern (see Figure 2a). The combined and adaptive use of the two strategies on the other hand is hypothesized to result in two-phase response-time pattern that, due to the application of the addition strategy, initially exhibits an increase in response times as a function of the numerosity of blocks in the grid, followed by a linear decrease in response times as soon as the individual switches to the subtraction strategy (see Figure 2b).

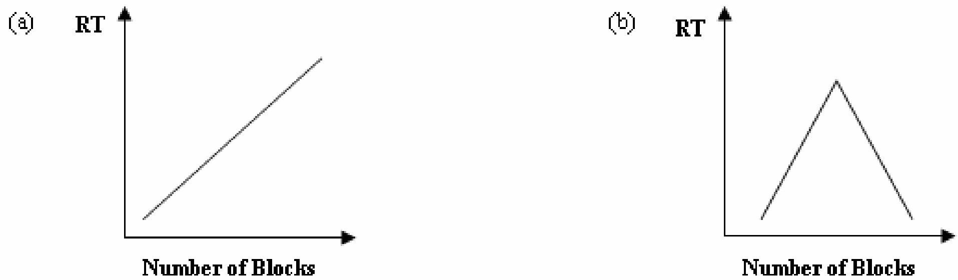

Figure 2: Hypothetical response-time patterns for respectively the addition strategy (a) and the addition and subtraction strategy (b).

This numerosity judgment task has two interesting features that make it particularly useful for testing a variety of hypotheses regarding the selection and execution of people's strategies. First, the associative strength of the two strategies--which is based on the (projected) speed and accuracy of each strategy on a particular problem--is assumed to vary gradually as a function of the numerosity of coloured blocks in the grid. That is, the addition strategy becomes less suitable as the number of blocks in the grid increases, whereas the reverse holds for the subtraction strategy. This feature allows assessing participants' strategy adaptivity at the level of a single trial. A second interesting feature is that the two strategies require a different amount of cognitive resources for their execution and thus vary in their degree of difficulty. Indeed, following our rational task analysis, one can assume that the subtraction strategy is cognitively more demanding than the addition strategy, not only because doing subtractions is more difficult than doing additions, but also because a proper application of the subtraction strategy involves executing the following steps: (a) determination of the grid size, (b) determination of the number of empty squares, and (c) subtracting the number of empty squares from the total number of squares in the grid. In contrast, application of the addition strategy involves only one step which is in terms of its nature and complexity similar to step (b) of the subtraction strategy, namely determining the number of coloured blocks in the grid. As a result of this difference in required mental effort, it can be assumed that the change point will not be located at the mathematical midpoint of the numerosity range (*i.e.*, the smallest trial wherein the number of blocks exceeds the number of empty squares (e.g., 51 in a 10 x 10 grid) but on a somewhat larger numerosity. Stated differently, the location of the change point will not be solely based on the objective task characteristics, but it will also depend on participants' own evaluation of when the more complex subtraction strategy becomes advantageous over the addition strategy in speed and/or accuracy.

Method for Strategy Identification

People's strategies for solving the above-mentioned task can be identified by fitting their observed response-time patterns with the hypothetical response-time patterns presented in Figure 2 by means of two regression models: the simple linear model and the two-phase segmented linear regression model (Beem, 1993, 1995). In the well-known linear models, the relationship between the independent and the dependent variable is described by a linear regression equation of the form $Y = a + bx + e$, where x is the independent and y is the dependent variable, the parameters a and b denote respectively the intercept and the slope of the regression lines and e is the error term. The two-phase segmented linear regression model looks for a 'break' or 'change point' in the data and accordingly computes two regression equations, which hold for different ranges of the independent variable. The two-phase segmented model can be formally described as:

$$Y = a_1 + b_1x + e \ (x \leq s)$$
$$Y = a_2 + b_2x + e \ (x > s)$$

Besides the parameters x, y, a_i (i = 1,2), b_i (i = 1,2) and e, which are the same as for the simple linear regression model, the two-phase segmented model has an additional parameter, s, which is called the 'change point' or 'break point'. For values of the independent variable up to s the first regression equation is fitted, while for values larger than s the second equation--with a different intercept and slope--holds. Although the two-phase segmented linear regression model is less well known and used, it is, according to Beem (1993, 1995; Ippel & Beem, 1987), ideally suited for the study of strategy shifts like those involved in the present research program. Indeed, the change point that is computed by the segmented linear regression model identifies the trial on which one strategy is replaced by the other. Consequently, this procedure enables an identification of the different strategies based on their specific properties as well as a determination of their range of application.

The two hypothetical response-time patterns from Figure 2 can be defined in terms of the different parameters of the statistical models presented above. This leads to the following characterisation of each pattern:

1. Pattern 1 (always addition): no change point, and the only b-parameter is positive.
2. Pattern 2 (first addition, then subtraction): one change point, a positive b_1-parameter and a negative b_2-parameter.

The strategies can be identified by going through the following stepwise procedure. First, the two-phase segmented linear regression model is fitted to all individual response-time patterns. Next, the data are tested for the occurrence of a change point following the *cusums* method (Brown, Durban, & Evans, 1975; Schweder, 1976). When the cusum test has detected a change point in the individual response-time pattern, this data pattern is further tested for a possible fit with Pattern 2 by computing significance tests regarding the linear restrictions of the b-parameters from the different regression equations by using the F-type statistic (Beem, 1993). When we do not detect a change point at all, we assume that the response-time pattern of that particular subject is similar to Pattern 1 and, thus, that only the addition strategy is used.

After having outlined the paradigm and its corresponding rational task analysis as well as the method for strategy identification, we now will present three studies in which we tested the usefulness of Lemaire and Siegler's (1995) theoretical framework for analyzing different aspects of participants' strategy use in the context of this numerosity judgement task. These aspects include: (a) the development of numerosity judgement strategies (Study 1), (b) the influence of intelligence and feedback on the strategy (Study 2) and, (c) the effect of contextual variations on the use of these strategies (Study 3).

Study 1: The Development of Children's Numerosity Judgment Strategies

Background

In this study, we applied Lemaire and Siegler's (1995) theoretical framework together with the choice/no-choice method (Siegler & Lemaire, 1997) to investigate the development of children's strategy use in the aforementioned numerosity judgment task. We relied on a cross-sectional design in which we compared the strategic performance of three different age groups (second graders, sixth graders and university students) on the four parameters of strategy competence by means of the choice/no-choice method. The application of this method resulted in three conditions: (a) a *choice* condition wherein participants were free to use either the addition or the subtraction strategy on each problem of the task, (b) a *no-choice addition* condition in which they had to apply the addition strategy on all problems of the task, and (c) a *no-choice subtraction* condition in which they had to use the subtraction strategy on all problems.

Based on the above-mentioned rational task analysis and its direct implications we formulated two sets of predictions. A first set of predictions concerned strategic aspects of numerosity judgments. First, we expected an unequal distribution of the use of both strategies, with the addition strategy being used more frequently than the subtraction strategy. Second, we expected that, under no-choice conditions, the addition strategy would be faster and more accurate than the subtraction strategy. The second of predictions was related to age-related differences in strategic competence. Taking into account that younger participants have fewer working memory resources and less practiced arithmetic skills than older participants, we first expected that not all participants from the youngest each group would use the subtraction strategy in the choice condition. Second, we predicted that the frequency of use of the subtraction strategy would increase with age, whereas the frequency of use of the addition strategy would decrease. Third, there would be an age-related increase in the efficiency of both strategies. Finally, the adaptivity of strategy choices would increase with age, since participants become better in calibrating their strategy choices as they grow older.

Method

Participants from three different age groups were involved in this study: 25 third graders (8 – 9 yrs.), 20 sixth graders (11 – 12 yrs.), and 37 university students (21 yrs.). They were instructed to determine different numerosities of green blocks that were presented in a 7 x 7 grid as quickly and accurately as possible in three conditions. In the choice condition, they were allowed to choose freely between the addition or the subtraction strategy to determine all numerosities from 1 to 49. In the no-choice addition condition, participants were required to determine the numerosities from 1 to 42 by means of the addition strategy, whereas in the no-choice subtraction condition participants were asked to determine the numerosities from 8 to 49 by using the subtraction strategy.[2] The presentation order of the different trials within each condition was randomized across participants. Furthermore, the order of the conditions was counterbalanced across participants with the important restriction that the choice condition was always presented first, so that strategy choices in the choice condition could not be influenced by recency effects.

On each trial, participants were requested to point to the units they were counting. This enabled the experimenter to unambiguously identify their strategy use on each trial in the choice condition, whereas it ensured that participants only used the required strategy in the no-choice conditions. The stimuli remained on the screen until participants had made their numerosity judgment. They were asked to verbally state their answer as soon as they knew it. The experimenter then immediately pressed a key that stopped the computer timer and emptied the grid. After the response was typed in by the experimenter, a new stimulus appeared on the screen. After each trial, the computer recorded participants' response and response time (with an exactitude of 0.1 s). There was no limit on the presentation time of the different problems. Before the start of the actual experiment, participants solved five example trials that were representative of the whole continuum of numerosities in the grid.[3]

Measuring Strategy Adaptivity at an Individual Level

The choice/no-choice method allows analyzing the adaptivity of strategy choices by assessing the extent to which participants select their strategies as a function of the unbiased estimates of each strategy's performance on the respective problems of the task. Based on the procedure for strategy identification outlined before, we developed a technique for measuring this strategy adaptivity on an individual level. According to the rational task analysis, the adaptive application of the addition and subtraction strategy in the choice condition would yield a two-phase response-time pattern similar to the hypothetical pattern in Figure 2b. As outlined earlier, the application of the two-phase segmented linear regression model on the individual response-time patterns yielded by the choice condition allows determining the *observed* change point.

In addition to this observed change point, it is also possible to derive an *optimal* change point based on the response times of the two no-choice conditions. This optimal change point can be located by running a simple linear regression model on the individual response-time patterns of both the no-choice addition and the no-choice subtraction condition. A plot of both regression equations in a single graph yields two regression lines that intersect each other. Each regression line represents an unbiased estimate of the speed of each of both strategies. As a consequence, the intersection of both regression lines demarcates the optimal change point, *i.e.*, the trial on which the subtraction strategy becomes faster than the addition strategy without being less accurate, at least when the accuracy of the responses is kept under control. Since the optimal change point indicates for each individual the trial on which it is most efficient to switch from the addition strategy towards the subtraction strategy, we consider a subject who switches to the subtraction strategy on this trial (in the choice condition) as being perfectly adaptive. As a consequence, the absolute difference in location between the observed and the optimal change point can be conceived as a measure of adaptivity: the closer both types of change points are located to each other, the better an individual's strategy choices are calibrated to his/her unbiased estimates of strategy performance. In other words, the *smaller* this difference, the *more* adaptive the strategy choices of the subject (see Figure 3).

[2] We did not test the upper end (i.e., numerosities 43-49) of the numerosity continuum in the addition condition and the lower end (i.e., numerosities 1-7) of the numerosity continuum in the subtraction condition because previous research has shown that the response-time patterns of the no-choice conditions exhibited a break in these ends. Observations of participants' overt strategic behaviour clearly showed that such a break was caused by the fact that when there were many green blocks (in the case of the addition condition), or many empty squares (in the case of the subtraction condition), some participants started to count by (full) rows instead of continuing to count one by one or in small groups of 2-5 squares.

[3] For a detailed description of the method, we refer to Luwel, Lemaire, and Verschaffel (2005).

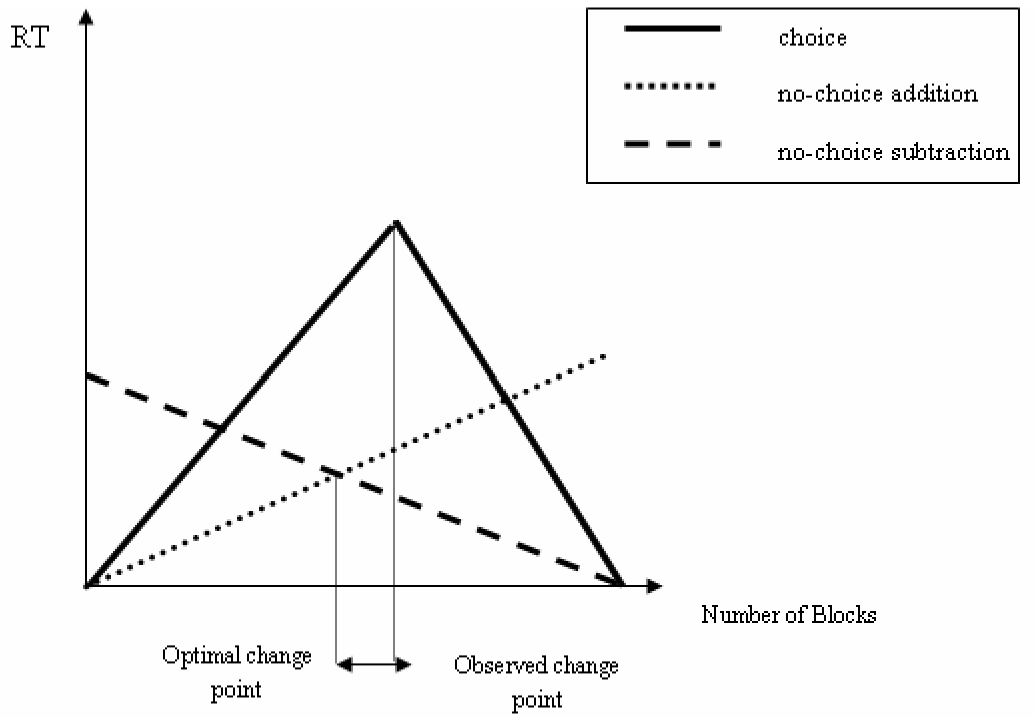

Figure 3: Schematic presentation of the difference between the 'observed' and the 'actual' change point.

As an example, suppose that Participant X as well as Participant Y switched towards the subtraction strategy on a trial with 27 blocks. At first sight, one might conclude that both participants were equally adaptive in their strategy choices. However, let's assume that the data of the no-choice conditions indicate that Participant X has an optimal change point of 28, whereas the optimal change point of Participant Y is 32. Based on these unbiased estimates of strategy performance from the no-choice conditions, one can conclude that Participant X is more adaptive in his strategy choices than Participant Y since his strategy choices are more finely calibrated towards his unbiased estimates of strategy performance as indicated by the absolute difference between both types of change points.

Results

Strategy repertoire. All sixth graders and adults applied both the addition and subtraction strategy in the choice condition, whereas only 60% of the third graders used the two strategies. This means that 40% of the youngest participants solely used the addition strategy to solve all trials in the choice condition.

Strategy frequency. A one-way analysis of variance with age (third, grade, sixth grade, and adults) as between-subjects factor was carried on the percentage of trials on which the subtraction strategy was applied. This analysis showed a significant effect of age, $F(2, 79) = 32.96$, $p < .0001$. We found that the adults ($M = 36\%$) and the sixth graders ($M = 33\%$) applied the subtraction strategy significantly more frequently than the third graders ($M = 12\%$).

Strategy efficiency. Strategy efficiency was analyzed in terms of speed (*i.e.*, solution times) and accuracy (*i.e.*, error rates measured as the absolute difference between the given response and the correct answer). We will only describe the strategy efficiency results from the no-choice conditions since only these conditions yielded unbiased measures of strategy speed and accuracy. ANOVAs of no-choice mean solution times and error rates were run with 3 (Group: adults, sixth, and third graders) x 2 (Strategy: addition vs. subtraction) designs, with age as the only between-subjects factor. Only solution times of problems that were solved correctly were included in our analysis.

For the solution times, we observed significant main effects of age, $F(1, 79) = 52.40$, $p < .0001$, and strategy type, $F(1, 79) = 269.91$, $p < .0001$. Both variables were involved in a significant interaction, $F(2, 79) = 36.74$, $p < .0001$ (see Figure 4). The different age groups differed from each other with respect to the speed of the subtraction strategy (*M*s: 21.4 s, 16.9 s, and 12.5 s for the third graders, sixth graders and adults, respectively), whereas for the addition strategy there was only a significant difference between the adults ($M = 10.5$ s) and third graders ($M = 14.3$ s). Moreover, the addition strategy was significantly faster than the subtraction strategy in all age groups.

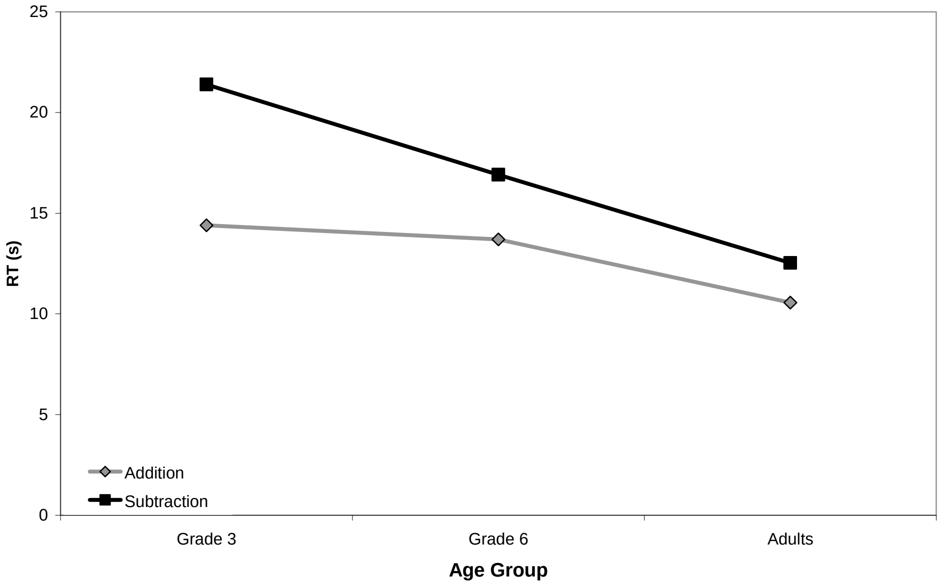

Figure 4: No-choice solution times of the addition and subtraction strategy for each of the three age groups.

For the error rates, we also found significant main effects of age, $F(2, 79) = 13.75$, $p < .0001$, and strategy type, $F(1, 79) = 39.71$, $p < .0001$. We also observed a significant age x strategy interaction, $F(2, 79) = 9.43$, $p = .0002$ (see Figure 5). Third graders ($M = 1.87$) were significantly less accurate than the sixth graders ($M = 0.90$) or the university students ($M = 0.39$) in their execution of the subtraction strategy, whereas there was no significant difference in accuracy between the last two age groups. We did not find any significant differences among the three age groups with respect to the accuracy of the addition strategy Third and sixth graders were significantly more accurate in executing the addition than the subtraction strategy (*M*s: 0.48 vs. 1.87 and 0.18 vs. 0.39, for respectively third and sixth graders), whereas adults were equally accurate in their execution of both strategies.

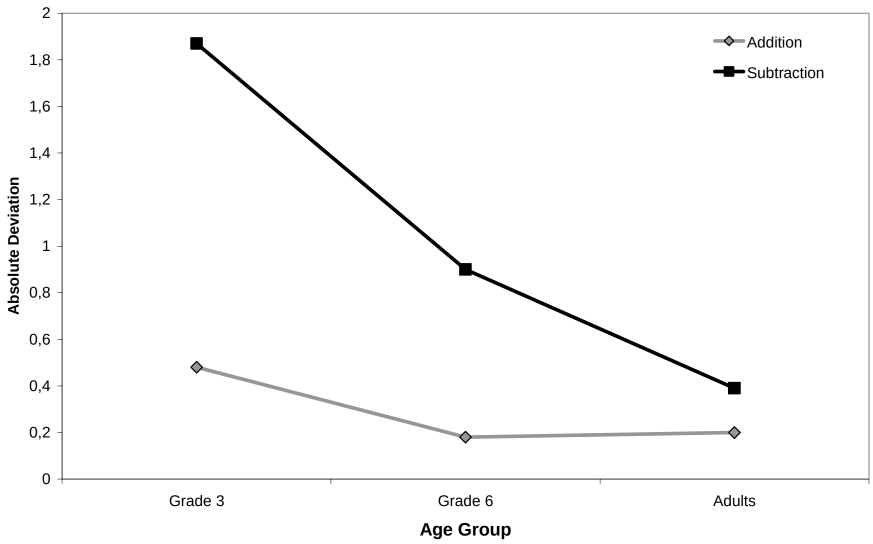

Figure 5: No-choice error rates of the addition and subtraction strategy for each of the three age groups.

Adaptivity of strategy choices. As outlined before, the absolute distance between the actual and the projected change point can be considered as a measure of adaptivity. To examine whether this adaptivity would change with age, we determined for each participant who had applied the two strategies in the choice condition, the observed as well as the optimal change point and we calculated the absolute difference between these change points. The means and *SD*s of the actual and projected change points as well as the absolute difference between these change points are displayed in Table 1 for each age group separately.

Table 1: Means and Standard Deviations of Both Types of Change Points and their Absolute Difference.

	Observed Change Point		Optimal Change Point		Absolute Difference	
Age Group	**M**	**SD**	**M**	**SD**	**M**	**SD**
Grade 3	39.67	6.28	29.93	1.83	10.67	5.21
Grade 6	33.16	5.79	24.16	1.46	9.11	5.93
Adults	29.65	4.60	27.22	1.36	4.05	2.88

Note. The fact that mean of the absolute differences differs slightly from the difference between the means of both types of change points is due to the presence of some negative differences between both types of change points.

A one-way ANOVA with age as the only between-subjects variable was conducted on the absolute difference scores between the two change points. This analysis revealed a significant main effect of age, $F(2, 68) = 14.36$, $p < .0001$. It was found that the absolute distance between the two change points was significantly smaller in adults than in sixth graders or in third graders. Moreover, the absolute distance between the two types of change points was marginally significantly smaller in sixth graders than in third graders ($p = .06$).

Discussion

The present study showed the usefulness of Lemaire and Siegler's (1995) theoretical framework for analyzing children's strategy development in a task in which it had not been applied previously. We found that there were four sources of increased performance with age in the present numerosity judgment task. A first source of improvement was the acquisition of the insightful, but at the same time, more complex subtraction strategy in children's strategy repertoire. As outlined in the rational task analysis, a proper application of this strategy yields faster and more accurate answers in the upper range of the numerosity continuum which makes it useful to incorporate this strategy in one's strategy repertoire. Second, we observed an increase in the frequency of use of this subtraction strategy between third and sixth grade. As children grow older, they gradually extend the range of application of this cognitively more demanding strategy towards smaller numerosities which allows a further improvement in speed and accuracy on this task. The extension of the range of application is also evidenced by the location of the change points in the different age groups. A third source of improvement with age is a more efficient execution of both strategies and especially of the subtraction strategy. This presumably reflects an increase in working memory resources and/or improved arithmetic skills which enables participants to trigger and execute each cognitive operation within a strategy more quickly. Obviously, this effect will even be more pronounced on strategies involving more steps and cognitive resources, such as the subtraction strategy. A last source of improved overall performance in children's numerosity judgment was an increased adaptivity of strategy choices with age: As they grew older, participants used more and more each strategy when it works best for them on a particular problem. This resulted in increased choice benefits with age. The age-related changes found here are in line with results of previous empirical studies in many cognitive domains such as mental arithmetic (Lemaire & Siegler, 1995), spelling (Lemaire & Lecacheur, 2002a) or, computational estimation (Lemaire & Lecacheur, 2002b), as well as with simulations of the development of children's strategy choices such as the SCADS-model (Shrager & Siegler, 1998).

The Effect of Intelligence and Feedback on the Selection and Execution of Numerosity Judgement Strategies

Background

In our second study, we examined the role of intelligence on the four parameters of strategic performance. In addition, we investigated whether the provision of (different types of) feedback resulted in improvements in one or more of these parameters and, whether this effect of feedback differed as a function of the level of intelligence. Of course, many studies have already examined the effect of intelligence on children's strategic competence, but they never looked at all four strategic parameters at the same time (e.g., Gaultney, Bjorklund, & Goldstein, 1996; Geary & Brown, 1991). With respect to feedback, we wanted to test the effect of two different types of feedback, namely *strategy feedback* (SFB), which informed students about the appropriateness of their strategy choice on each trial and *outcome feedback* (OFB) which provided the children with information about the correctness of their responses (Kluger & De Nisi, 1996; Smith & Ragan, 1993).

We conducted an experiment in which children of three different intelligence levels (low, average and high) were asked to determine different numerosities of green blocks that were presented in a 7 x 7 grid. All children solved the experimental task in three sessions: a pre-test, an intervention and a post-test session. The choice/no-choice method was applied in the pre- and the post-test sessions in a similar fashion as in the Study 1. The intervention session only involved a choice condition in which half of the participants in each intelligence group received outcome feedback, while the other half received strategy feedback.

The following predictions regarding the intelligence-related differences on the four parameters of strategic competence were formulated. First, we expected that the number of children that would spontaneously use the subtraction strategy besides the addition strategy in the choice condition of the pretest would be positively related with intelligence. Second, we predicted that the frequency of the subtraction strategy in the choice condition of the pretest would also increase with intelligence. Third, we expected an intelligence-related improvement in the efficiency of both strategies in the no-choice conditions of the pretest: both strategies would be applied more quickly and accurately as intelligence increased. Fourth, we predicted higher levels of adaptivity with an increasing level of intelligence in the pretest. Two additional predictions were made with respect to the provision of feedback. First, since low intelligent children are assumed to benefit more from feedback than high intelligent children (Rohwer, 1973), we anticipated that the differences among the three intelligence groups on each of these four parameters would become smaller due to the provision of feedback. Second, we expected that strategy feedback would have a larger effect on children's strategic competence, and thus result in a more efficient strategy execution and a more adaptive strategy selection, than outcome feedback.

Method

Based on the mean full scale IQ of the standardized Greek version of the WISC-III, we selected 40 low intelligent (mean IQ = 77.37, SD = 4.25, range: 68-80), 40 average intelligent (mean IQ = 103.57, SD = 6.88, range: 90-110), and 40 high intelligent pupils (mean IQ = 128.67, SD = 5.98, range: 123-145) from a larger sample of 1689 pupils that attended the first grade of secondary school in Greece. In each group boys and girls were almost equally represented. Pupils were instructed to determine all numerosities of green blocks between 20 and 45 presented in a 7 x 7 grid as quickly and accurately as possible in all experimental sessions (pretest, intervention, posttest) and conditions (choice and no-choice conditions). The procedure was completely the same as in Study 1, except that in the intervention session, participants were given feedback at the end of each trial. Half of the participants in each intelligence group received outcome feedback (OFB) which informed them about the accuracy of their numerosity judgment in each trial (i.e., the number of blocks that their answer deviated from the actual numerosity), whereas the other half received strategy feedback (SFB), which informed about the appropriateness of their strategy choice on each trial as indicated by the no-choice data from the pre-test session (see further).[4]

Results

Strategy repertoire. Table 2 presents the number of children in each intelligence and feedback group that uses both the addition and subtraction strategy in the choice condition of the different sessions. As can be derived from this table, almost all high intelligent children applied the subtraction strategy spontaneously, whereas this was the case for only half of the average intelligent and none of the low intelligent children. Moreover, there was no difference between the two feedback groups in the pretest session. In the intervention session, there was a large increase in the number of low intelligent children with both strategies in their repertoire and this increase was more pronounced in the SFB group than in the OFB group. For the average intelligent children, we observed a strong increase in the SFB group only. All low and average intelligent children in the SFB group had incorporated the subtraction strategy in their strategic repertoire by the end of the intervention session. In the posttest session, the number of low and average intelligent children that used both strategies further increased in the OFB group, however, without reaching the maximum. In the high intelligence group, this maximum was already reached for both feedback groups in the intervention session.

Table 2: Percentage of participants in each intelligence and feedback group that used the addition and subtraction strategy in the choice condition of the pretest, intervention, and posttest session.

	Intelligence Level		
Feedback	**Low**	**Medium**	**High**
		Pretest	
Outcome	0%	50%	95%
Strategy	0%	50%	95%
		Intervention	
Outcome	60%	55%	100%
Strategy	100%	100%	100%
		Posttest	
Outcome	70%	80%	100%
Strategy	100%	100%	100%

Frequency of strategy use. A 3 (intelligence: low, average, and high) x 2 (feedback type: OFB vs. SFB) x 3 (session: pretest, intervention, and posttest) ANOVA with repeated measures on the last factor was conducted on the percentage subtraction strategy use in each of the three choice conditions.

The analysis showed significant main effects of intelligence, F (2, 114) = 41.04, p < .0001, feedback, F(1, 114) = 15.35, p = .0002, and session, F(2, 228) = 108.34, p < .0001. We also observed a significant intelligence x session interaction, F(4, 228) = 18.53, p < .0001 (see Figure 6). This interaction revealed that the high intelligent children already used

[4] For a detailed description of the method, we refer to Luwel, Foustana, Verschaffel, & Papadatos (in preparation).

the subtraction strategy very frequently during the pretest (M = 56%) and that they only showed a minor non-significant increase in their use of the subtraction strategy in the remainder of the experiment (*M*s: 62% and 64% for the intervention and posttest, respectively). The low and average intelligent children on the other hand, showed a significant increase in their subtraction strategy use from the pretest (*M*s: 0% and 25% for low and average intelligent children, respectively) to the intervention session (*M*s: 41% and 47%), after which they also showed a minor and non-significant increase (*M*s: 41% and 52%). Whereas the three intelligent groups differed significantly from each other in their subtraction strategy use during the pretest, we only observed a significant difference between the high and low intelligent children in the intervention and posttest. Finally, we observed a significant feedback x session interaction, $F(2, 228) = 7.74$, $p = .0005$ (see Figure 6). There was a significant increase in the use of the subtraction strategy between the pretest and the intervention in both feedback groups but this increase was more pronounced for the SFB group (M = 29%) than for the OFB group (M = 16%) resulting in a significantly higher use of the subtraction strategy during the intervention and posttest for the SFB group (*M*s: 58% and 61% for the intervention and posttest, respectively) compared to the OFB group (*M*s: 42% and 45% for the intervention and posttest, respectively).

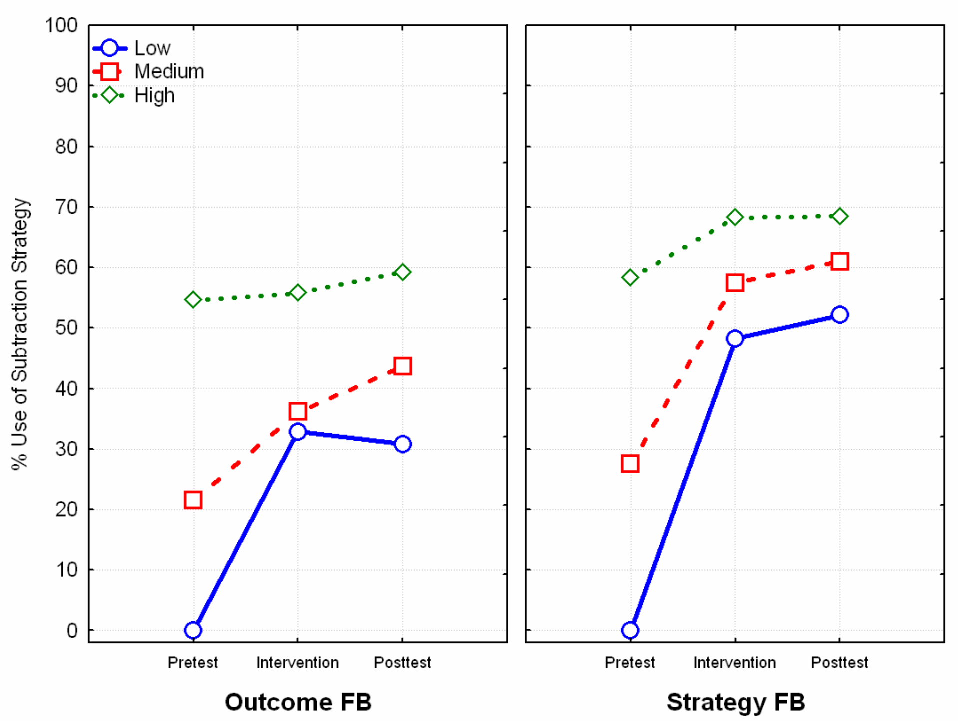

Figure 6: Percentage subtraction strategy use in the choice condition of each session for each intelligence and feedback group.

Strategy efficiency. Similar to the first study, strategy efficiency was analysed in terms of no-choice solution times and error rates as measured in the pre- and posttest. ANOVAs of no-choice mean solution times and error rates were run with 3 (intelligence: low, average, and high) x 2 (feedback: OFB vs. SFB) x 2 (session: pretest vs. posttest) x 2 (strategy: addition vs. subtraction) with repeated measures on the last two factors. Only solution times of problems that were solved correctly were included in our analysis.

For the solution times, we observed significant main effects of intelligence, $F(2, 114) = 33.04$, $p < .0001$, session, $F(1, 114) = 45.10$, $p < .0001$, and strategy, $F(1, 114) = 30.81$, $p < .0001$. Furthermore, we observed a significant intelligence x strategy interaction, $F(1, 114) = 49.67$, $p < .0001$ and a significant session x strategy interaction, $F(1, 114) = 58.51$, $p < .0001$. Both interactions were involved in a significant intelligence x strategy x session interaction, $F(2, 114) = 12.69$, $p < .0001$ (see Figure 7). This interaction showed that, during the pretest, the high and average intelligent children were significantly faster in their execution of the subtraction strategy than the low intelligent children (*M*s: 19.60 s, 13.40 s, and 10.03 s for the low, average and high intelligent children respectively). Although the low and average intelligent children showed a significant increase in subtraction strategy speed between the pre- and the posttest, the low intelligent children remained significantly slower in their execution of this strategy compared to their average and high intelligent peers (*M*s: 15.74 s, 11.07 s, and 8.91 s for the low, average and high intelligent children, respectively). There were no significant differences in the speed of the addition strategy, neither between intelligence groups nor between sessions.

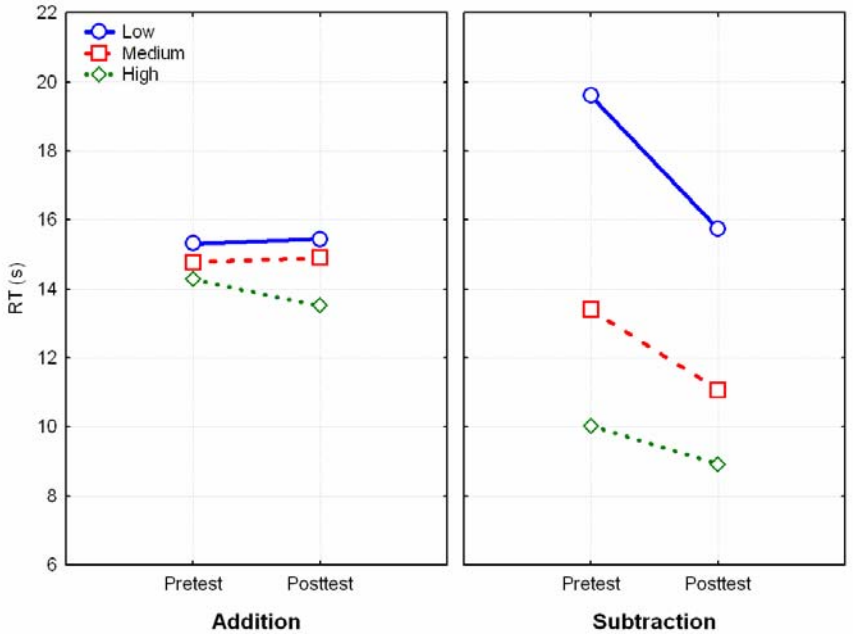

Figure 7: Mean solution times for the addition and subtraction strategy in the no-choice conditions of the pre- and posttest session for each intelligence group[5].

The results for the error rates revealed significant main effects of intelligence, $F(2, 114) = 21.58$, $p < .0001$, strategy, $F(1, 114) = 40.43$, $p < .0001$, and session, $F(1, 114) = 35.65$, $p < .0001$. We also observed an intelligence x strategy interaction, $F(2, 114) = 10.42$, $p < .0001$ (see Figure 8). The subtraction strategy was less accurate than the addition strategy in the low (*M*s: 1.08 vs. 0.46 for addition and subtraction, respectively) and average intelligence group (*M*s: 0.59 vs. 0.27) but not in the high intelligence group (*M*s: 0.24 vs. 0.20). Furthermore, it was found that the high intelligent children applied the subtraction strategy more accurately than the average intelligent children and the average intelligent children executed this strategy more accurately than the low intelligent children. However, we did not observe any differences between the intelligence groups regarding the accuracy of the addition strategy. Finally, there was a session x strategy interaction, $F(1, 114) = 12.95$, $p = .0004$ (see Figure 8). The low intelligent children ($M = 0.98$) were significantly less accurate during the pretest than the average ($M = 0.55$) and the high intelligent children ($M = 0.28$). Although the low and average intelligent children showed a significant increase in their overall accuracy between the pretest and the posttest, the low intelligent children remained significantly less accurate compared to the average and high intelligent children (*M*s: 0.57, 0.31, and 0.16 for the low, average and high intelligent children, respectively).

Adaptivity of strategy choices. Like in Study 1, we calculated the absolute difference between the observed and the optimal change point which were respectively derived from the individual choice and no-choice response-time patterns in the pre- and the posttest session. Next, a 3 (intelligence: low, average, and high) x 2 (session: pretest vs. posttest) x 2 (feedback type: OFB vs. SFB) ANOVA was run on these difference scores between. This analysis revealed significant main effects of intelligence, $F(2, 113) = 13.31$, $p < .0001$, feedback, $F(1, 113) = 7.51$, $p = .007$, and session, $F(1, 76) = 36.16$, $p < .0001$. Furthermore, we observed a feedback x session interaction, $F(2, 113) = 11.10$, $p = .001$ (see Figure 9). It was found that there was no difference in adaptivity between both feedback groups during the pretest (*M*s: 8.64 and 8.60 for OFB and SFB, respectively). Although both feedback groups showed a significant decrease in the distance between both types of change points from the pre- towards the posttest, this decrease was much more pronounced in the SFB group, resulting in a significant difference in adaptivity between both groups during the posttest (*M*s: 6.17 and 2.87 for OFB and SFB, respectively). Finally, we observed an intelligence x session interaction, $F(2, 113) = 6.44$, $p = .002$. In the pretest session, the high intelligent children ($M = 5.18$) were significantly more adaptive than the average ($M = 10.25$) and low intelligent ($M = 10.49$) children. In the posttest session, we found that the low and average intelligent students had made a significant improvement in adaptivity as a result of which the initial difference

[5] Given that Study 1 had shown that the addition strategy is faster than the subtraction strategy, it seems to be counter-intuitive that we observe here an inverse pattern of results. However, it is important to note that, contrary to Study 1, 76% of the problems in the present study included a grid that was more than half filled with green blocks. It is especially on these large-numerosity problems that the subtraction strategy is faster than the addition strategy.

with the high intelligent group had disappeared (*M*s: 4.79, 5.23, and 3.50 for the low, average, and high intelligent children, respectively). The high intelligent children only showed a slight (non-significant) increase in adaptivity from the pre- to the posttest.

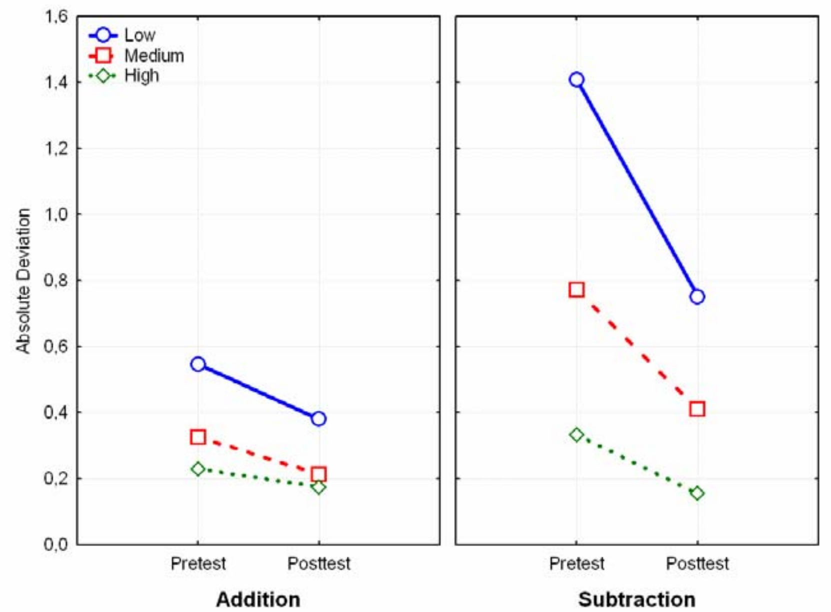

Figure 8: Mean error rates for the addition and subtraction strategy in the no-choice conditions of the pre- and posttest session for each intelligence group.

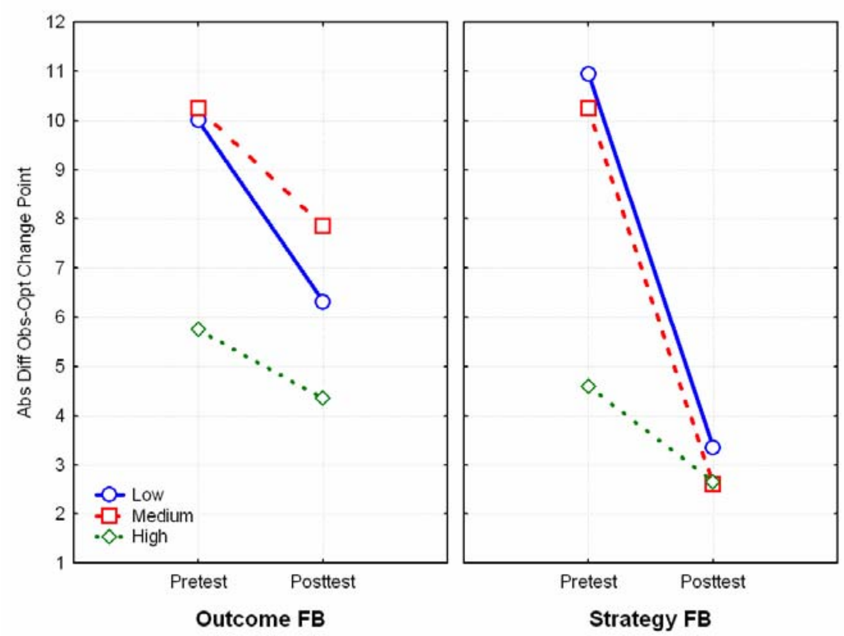

Figure 9: Absolute difference between the observed and the optimal change point in the pre- and posttest session for each intelligence and feedback group.

Discussion

The present study demonstrated that Lemaire and Siegler's (1995) theoretical framework can also be used for testing hypotheses about the impact of other subject variables than age on the four parameters of strategic performance, such as intelligence. In addition, it also enabled us to investigate the effect of two types of feedback on children's strategic performance.

The pretest data show a very large influence of intelligence on the different parameters of strategic competence. We observed that, with increasing intelligence, children are more prone to spontaneously use an insightful strategy, apply this strategy more frequently, execute it more efficiently and select their strategies more adaptively. Provision of feedback, however, results in a strong improvement in all these parameters. Medium and, especially, low intelligent children benefit most of the provision of feedback whereas providing feedback had no effect at all on the strategic performance of high intelligent children. This finding is in line with an assumption made by Rohwer (1973) who states that high intelligent children will profit less from the provision of feedback than children with a lower intelligence. The underlying rationale is that the selection and execution of strategies is considered to be more optimal in the former group than in the latter one, and thus that the learning gain from receiving feedback will become smaller as a function of the intelligence level. Although medium intelligent children benefited less from feedback than low intelligent children, it did enable them to reach the same levels of performance as high intelligent children. This was not the case for the low intelligent children for most of the parameters of strategic competence.

Another interesting observation was that providing feedback about children's strategy choices resulted in a larger improvement than informing them about the accuracy of their outcomes. Interestingly, this differential effect of feedback was only observed in those parameters of strategic competence that dealt with strategy *selection* but not with strategy *execution*.

Adaptation of Numerosity Judgment Strategies to Task Characteristics

Background

After having extended the applicability of Lemaire and Siegler's (1995) framework towards other subject variables than age, we now wanted to test whether it could also be applied to investigate the effect of contextual variables on peoples' strategic competence. According to Payne, Bettman, and Johnson (1993), the task context is, besides the characteristics of the strategy and the individual, one of the factors that affect people's strategy choices. These three factors are assumed to interact with each other when making an adaptive strategy choice. As a result, a given strategy can be regarded as relatively more effective than other strategies in one context and relatively less effective than these same strategies in another context.

The context variable that we manipulated in the present study was the diversity in grid sizes. All participants ran two different conditions: a *pure* condition and a *mixed* condition. In the pure condition, all numerosities of blocks were presented in the same grid, whereas in the mixed condition the numerosities of blocks were shown in grids of different sizes. Based on our rational task analysis, one can assume that, a single determination of the grid size is sufficient for properly applying the subtraction strategy throughout the pure condition. However, in the mixed condition, one needs to determine the grid size for each trial on which one wants to apply the subtraction strategy correctly. This reasoning leads to the basic hypothesis that the pure condition would favour the use of the subtraction strategy more strongly than the mixed condition.

The following four predictions were made regarding the different parameters of strategic competence under these two conditions. First, with respect to the repertoire of strategies, we expected that all (adult) participants would apply the addition and subtraction strategy in both conditions. For the frequency with which the different strategies are applied, it was expected that participants would use the subtraction strategy on a smaller number of trials in the mixed condition compared to the pure condition. The extra step of determining the size of the grid in the mixed condition when using the subtraction strategy will lead participants to apply the subtraction strategy less frequently. Concerning the efficiency of strategy execution, we expected that, with respect to speed, the subtraction strategy would be faster in the pure condition than in the mixed condition for the same range of numerosities. Since participants need to determine the grid size only once in the pure condition in order to apply the subtraction strategy correctly, they can save a significant amount of time in this condition compared to the mixed condition. Since it was assumed that the addition strategy would be executed in the same way in the two conditions, we did not expect a difference between the two conditions with respect to speed of this strategy. Regarding accuracy, we expected that the subtraction strategy would be less accurate in the mixed condition than for the same numerosities in the pure condition. The requirement of the repeated determination of the grid size in the mixed condition will increase the probability of making mistakes when computing the total number of squares in the grid. Moreover, participants may err when they decide not to determine the grid size and mistakenly take one grid size for another. As for the solution times, we expected no difference between conditions with respect to the accuracy of the addition strategy. We did not make any specific predictions with regard to the adaptivity of strategy choices, mainly because we did not apply the choice/no-choice method in the present study, which made it impossible for us to analyse this parameter in detail.

Method

Twenty-four university students with a mean age of 21 years took part in the experiment. Each participant ran two different conditions during two consecutive days. The order of the conditions was counterbalanced over subjects. In the mixed condition, participants were randomly presented all possible numerosities of blocks from the 7 x 7 grid. These 49 trials were mixed with 20 randomly chosen numerosities of blocks, each from the 6 x 6 and 8 x 8 grid, and 5 randomly chosen numerosities of blocks, each from the 5 x 5 and 9 x 9 grid, resulting in a total number of 99 trials. In the pure condition, all problems were presented in the 7 x 7 grid. To enhance the comparability between conditions with respect to the number of trials to be presented, all possible numerosities of blocks were presented twice in the pure condition, resulting in a total of 98 trials. Depending on the condition, participants were informed whether the size of the grid would be the same throughout the whole session (pure condition), or that the blocks would be presented in different grid sizes (mixed condition). Except for the above-mentioned differences, the procedure was completely the same as in the previous two studies.[6]

Results

The focus was on participants' strategy use for judging numerosities in the 7 x 7 grid. Therefore, only those numerosities that were presented in the 7 x 7 grid in the mixed condition were included in the analyses. To maximize the comparability of the data between conditions, only the numerosities from 1 to 49 that were presented first in the pure condition were taken into account.

Strategy repertoire. In the two conditions, all participants used the addition and the subtraction strategy to solve the task.

Frequency of strategy use. A *t*-test for dependent samples indicated that the change point was located on a trial with a larger numerosity in the mixed (M = 30.71) than in the pure condition (M = 26.17), $t(23) = 3.91$, $p = .0007$. Stated differently, the subtraction was applied on a significantly larger number of trials in the pure (M = 47%) than in the mixed condition (M = 38%).

Efficiency of strategy use. We conducted 2(condition: mixed vs. pure) x 2(strategy: addition vs. subtraction) repeated measures ANOVAs on participants' solution times and error rates. To guarantee that all data within the range of small numerosities were associated with the addition strategy and all data within the range of large numerosities were produced by the subtraction strategy, we only included those trials on which *all* participants applied the addition and subtraction strategy, respectively. More specifically these were the trials from 1 to 17 for the addition strategy and from 40 to 49 for the subtraction strategy.

Figure 10 shows the pattern of mean solution times in both conditions. A visual inspection of this data pattern suggests that the mean solution times for the large numerosities (*i.e.*, right of the dotted line in Figure 10) are indeed larger in the mixed than in the pure condition, whereas there is no difference between conditions with respect to the solution times for the small numerosities (*i.e.*, left of the dotted line).

The ANOVA on the solution times revealed significant main effects of condition, $F(1, 23) = 60.70$, $p < .0001$, and strategy type, $F(1, 23) = 31.00$, $p = .0002$. As expected, both variables were also involved in a significant interaction effect, $F(1, 23) = 116.75$, $p < .0001$. The subtraction strategy was significantly slower in the mixed condition (M = 5.38 s) than in the pure condition (M = 2.73 s), whereas there was no difference between conditions for the speed of the addition strategy.

The ANOVA on the deviation scores showed main effects of condition, $F(1, 23) = 6.65$, $p = .02$, and strategy type, $F(1, 23) = 6.58$, $p = .02$. In line with our prediction, we also observed a significant interaction between both variables, $F(1, 23) = 8.02$, $p = .01$. The subtraction strategy was significantly less accurate in the mixed (M = 0.46) than in the pure condition (M = 0.04), whereas there was no difference between conditions for the addition strategy.

Discussion

The present study showed how variations in the context of a task can affect people's strategy choices. The extra time-consuming and cognitively demanding step of determining the grid size in the mixed condition made the subtraction strategy less fast and accurate than in the pure condition, as evidenced by lower efficiency scores for this strategy in the mixed condition. This resulted in a decline of the attractiveness of the subtraction strategy compared to the pure condition. Due to this reduced attractiveness of the subtraction strategy in the mixed condition, participants also applied this strategy in that condition on a smaller range of numerosities. As indicated by the location of the change points in the two conditions, they only applied the subtraction strategy on these trials for which it became more advantageous than the addition strategy. These results are in line with the theory of Payne *et al.* (1993) in which it is stated that the properties of the context affect the relative advantages and disadvantages of the various strategies available in ones strategy repertoire. Therefore, subjects will adapt their strategy choices to these contextual variations.

[6] For a detailed description of the method, we refer to Luwel, Verschaffel, Onghena, & De Corte (2003).

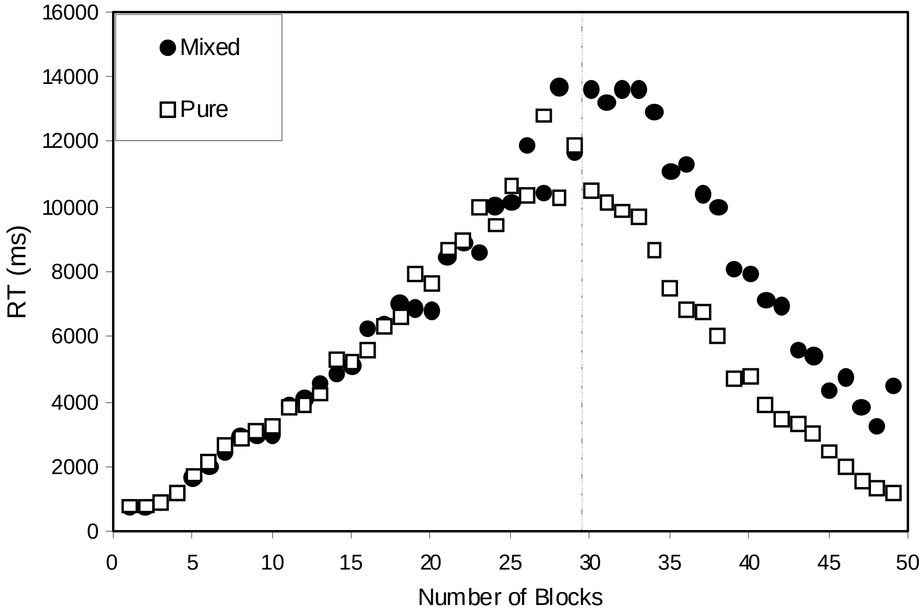

Figure 10: Mean solution times in the pure and the mixed condition.

General Discussion

The present article demonstrates the potential of Lemaire and Siegler's (1995) theoretical framework regarding the four parameters of strategic performance together with the choice/no-choice method for analysing different aspects of people's strategic performance. We have shown that this framework and its associated method are also apt to investigate developmental changes in a task domain in which it had not been used before, namely numerosity judgment. Furthermore, both can be used to examine the contribution of other subject variables than age on these four parameters. Finally, the framework has been proven to be helpful in checking which factors of strategic competence are affected by variations in task context.

Our results regarding the different aspects of the numerosity judgment strategies presented here are in line with more general findings from the cognitive strategy literature demonstrating that participants' strategy choices are dependent on the characteristics of the items, the context and the individual (Siegler, 1996; Verschaffel, Luwel, Torbeyns, & Van Dooren, in press). More specifically, with respect to item characteristics, we observed that participants applied each of the two strategies on those items where they were the most advantageous (*i.e.*, where they lead fastest to an accurate answer), namely the addition strategy was mainly applied on the items in the lower range of the numerosity continuum, whereas the subtraction strategy was primarily used on the items in the upper range. Second, participants adapted their strategies towards variations in the task context. Indeed, we observed that they applied the subtraction strategy less frequently if the context made the application of this strategy more difficult. Finally, there were also large influences of person characteristics like, age or intelligence on strategy choices. Participants applied the cognitively more demanding subtraction strategy more frequently and more efficiently with increasing age and intelligence.

Two important remarks need to be made with respect to the strategies being investigated in the present series of studies. First, the addition and subtraction strategy are not the only strategies that can be applied within this task. In some of our of studies (Luwel & Verschaffel, 2003; Luwel, Verschaffel, Onghena, & De Corte, 2000, 2001; Verschaffel, De Corte, Lamote, & Dherdt, 1998) we have found that, a number of participants applied a third strategy besides the addition and subtraction strategy, the so-called estimation strategy. This strategy is characterised by rather imprecise numerosity judgments, by relatively short solution times that are not seriously affected by the numerosity to be determined and is applied mainly in the middle range of the numerosity continuum. This estimation strategy can be considered as some kind of back-up strategy to which participants can resort if they do not possess the necessary time, motivation and/or knowledge and skills to determine the numerosities in (especially) the middle range.

Second, there exist several variants of the addition and the subtraction strategy. For instance, when applying the addition or subtraction strategy one can determine the number of green blocks/empty squares by counting them in groups of the same size (e.g., counting one by one, two by two, three by three,…) or one can divide them first in subgroups of different sizes after which one determines the number of units in each subgroup by means of a subitizing, counting or estimation procedure and add this result to a running total to arrive at a final numerosity judgment. Furthermore, there is one specific variant of the subtraction strategy in which one counts the number of empty squares down one by one from the total number of blocks in the grid instead of subtracting the total amount of empty squares from the total. We have recently terminated a study in which we explored the different variants of the addition and subtraction strategy in greater detail (Frickel, Luwel, Verschaffel, & Onghena, in preparation). In our future research, we will rely on online data-gathering techniques such as the recording of eye-movements to obtain a clearer picture of the exact nature of the variants of these strategies.

As we have demonstrated in this article, a proper analysis of an individual's strategic competence in terms of its four parameters requires the application of the choice/no-choice method. Although this method has made it possible to obtain unbiased measures of a strategy's efficiency and to assess the adaptivity of strategy choices in an appropriate way, this method does not come without hazards. First, this research method is labour-intensive and time-consuming, since participants need to run at least three conditions (one choice condition and two no- choice conditions). Evidently, this practical problem becomes greater when allowing more than two strategies in the choice condition. This is why we, and other researchers, tried to restrict the number of strategies under consideration. However, by reducing this strategy variety, one runs the risk of losing external validity. Second, the choice/no-choice method can only be employed with respect to tasks in which the experimenter can effectively control participants' strategy use. In other words, the experimenter must be assured that participants really do what they are requested to do in the no-choice conditions and that they only use the allowed strategies in the choice condition. This is why we requested participants to point on the computer screen the units they were counting in all our studies with the choice/no-choice method. Consequently, this method will be easier to use on tasks in which overt behaviour can be used to validate strategy use than on tasks in which it cannot. For an in-depth discussion of the strengths and weaknesses that are associated with the choice/no-choice method, we refer to Luwel, Onghena, Torbeyns, Schillemans, and Verschaffel (in press).

Although the present approach allowed us to study in a detailed way the development of children's numerosity judgment strategies, it does not allow us to draw strong conclusions about the discovery and early development of the subtraction strategy. This is due to the fact that the density of observations in such a cross-sectional design is too low to provide an answer to these specific research questions. A means for obtaining this kind of fine-grained information is the microgenetic method (Siegler, 2006; Siegler & Crowley, 1991). This method emphasizes high-density sampling of changing competence while the change is occurring. A common form of the method involves accelerating the change process by providing participants with experiences intended to promote discovery of more advanced concepts, strategies, rules, or theories. The increased density of opportunities to discover and exercise the new understanding allows more detailed examination of change than would otherwise be possible. The application of the microgenetic method to study the emergence and early development of the subtraction strategy for doing numerosity judgments is described in Luwel, Siegler, and Verschaffel (2008).

The present set of studies showed that the flexible and adaptive use of cognitive strategies can lead to large improvements in overall task performance. Many current curriculum reform documents, innovative curricula, textbooks, software, and other instructional materials based on these reform documents worldwide, stress the value of striving for such strategy flexibility/adaptivity, also in domains in which children have to judge numerosities such as counting or estimation. Future studies should find out which instructional contexts can play a role in achieving this goal by carrying out design experiments. Certainly these learning environments should be more powerful than the mere provision of feedback like in the intervention sessions of Study 2. Some basic features of this experimental environment aimed at achieving such a flexibility/adaptivity (besides the inevitable development of procedural skills) will be: (a) the integrated development of the different types of knowledge that we have mentioned in our rational task analysis, namely procedural, conceptual and metacognitive knowledge, (b) the development of positive beliefs and attitudes towards variety and flexibility in strategy use and reflection upon the conditions under which one strategy is more favourable than another (instead of beliefs and attitudes that only value perfect mastery of one routine procedure taught by the teacher) (c) the creation of a classroom culture that does not only value speed and accuracy but also the cleverness, simplicity and originality of a strategy (see Verschaffel *et al.*, in press).

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